Math/Mgmt 3502 (Ng/Fall 2007)
Problems for Queueing Models
For class on December 11-13, 2007.
Not due, but you should know how to do these;
I shall do the even numbered problems

1. A queueing system can hold no more than 4 customers. The arrival rate is $\lambda = 10$ per hour and the departure rate is $\mu = 5$ per hour. Both rates are independent of the number in the system n. Assume that the arrival and departure processes follow a Poisson distribution. Draw the complete transition diagram; then determine the following:
   a. The set of balance equations describing the system.
   b. The steady-state probabilities.
   c. The expected number in the system, L_s.
   d. The effective arrival rate, λ_{eff}.
   e. The expected waiting time in the queue, W_q.

2. Repeat the previous problem for a simplified single-queue model in which the service mechanism that allows only one customer in the system. Customers who arrive while the facility is busy will leave and never return. Assume that customers arrive according to a Poisson distribution and mean λ per unit and that the service time is exponential with mean value equal to $1/\mu$ time units.

3. A fast-food joint, Mickey-D, has one drive-in window. It is estimated that cars arrive according to a Poisson distribution at the rate of 2 every 5 minutes and that there is enough space to accommodate a line of 10 cars. Other arrivals can wait outside this space, if necessary. It takes 1.5 minutes on the average to fill an order, but the service time actually varies according to an exponential distribution. Determine the following:
   a. The probability that the facility is idle.
   b. The expected number of customers waiting but currently not being served.
   c. The expected waiting time until a customer can place his order at the window.
   d. The probability that the waiting line will exceed the capacity of the space leading to the drive-in window.

4. Cars arrive at a toll gate on a freeway according to a Poisson distribution with mean 90 per hour. Average waiting time for passing through the gate is 38 seconds. Drivers complain of the long waiting time. Authorities are willing to decrease the passing time through the gate to 30 seconds by introducing new automatic devices. This can be justified only if under the old system the number of waiting cars exceeds 5. In addition, the percentage of the gate's idle time under the new system should not exceed 10%. Can the new device be justified?

5. Customers arrive at a one-window drive-in bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. There are three spaces in front of the window, including the one for the car being served. Other arriving cars can wait outside these three spaces.
   a. What is the probability that an arriving customer can enter one of the three spaces in front of the window?
   b. What is the probability that an arriving customer will have to wait outside the three spaces?
   c. How long is an arriving customer expected to wait before starting service?
   d. How many car spaces should be provided in front of the window so that an arriving customer can wait in front of the window at least 20% of the time?

6. Return to the Mickey-D problem. Solve the entire problem if we assume that customers who cannot join the line in front of the service window will normally go elsewhere like KFC.

7. The MCL cafeteria can seat a maximum of 50 people. Customers arrive in a Poisson stream at the rate of 10 per hour. They are serviced at the rate of 12 per hour. For simplicity sake, assume that customers are serviced one at a time by one waiter.

   a. What is the probability that the next customer will not eat in the cafeteria because it is full?

   b. Suppose that 3 customers (with random arrival times) would like to be seated together. What is the probability that their wish cannot be fulfilled? (Assume that arrangements can be made to seat them together as long as there are three empty seats anywhere in the cafeteria.)

8. Patients arrive at a clinic (with one doctor) according to a Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour.

   a. What is the effective arrival rate at the clinic?

   b. What is the probability that an arriving patient will not have to wait?

   c. What is the expected waiting time until a patient is discharged from the clinic?

9. The LL Peanut mail-ordering facility has a single telephone line with provisions to keep at most three additional customers on hold until the operator is ready to take their orders. Calls arrive according to a Poisson stream every 5 minutes. The time needed to take each order is exponential with an average of 6 minutes.

   a. On the average, how long does a customer wait before being serviced by the operator?

   b. In your estimation, is the waiting time in part (a) reasonable for a facility of this type?

   c. Assuming that the facility will continue to use one telephone line only, can you suggest a way for reducing the "on hold" waiting time?

10. In an $(M/M/2) : (GD/\infty/\infty)$ queueing system, mean service time is 5 minutes and mean interarrival time is 8 minutes.

    a. What is the probability of a delay?

    b. What is the probability of at least one of the servers being idle?

    c. What is the probability that both servers are idle?

11. The computer center at UMM is equipped with three digital computers, all of the same type and capability. The number of users in the center at any time is equal to 10. For *each* user, the time for writing and inputting a program is exponential with mean rate .5 per hour. The execution time per program is exponentially distributed with mean rate of 2 per hour. Assuming that UMM's computer center is in operation on a full-time basis, and neglecting the effect of computer downtime, find the following.

    a. The probability that a program is not executed immediately upon receipt at the center.

    b. The average time until a program is released from the center.

    c. The average number of programs awaiting execution.

    d. The expected number of idle computers.

    e. The percentage of time the computer center is idle.

    f. The average percentage of idleness *per computer*.

12. An airport terminal services 3 types of customers: those arriving from hick areas, those arriving from suburban areas, and the transit customers who are changing planes at the airport. The arrivals distribution for each of the three groups is assumed to be Poisson with mean arrival rates 10, 5, and 7 per hour, respectively. Assuming that all customers require the same type of service at the terminal and that the service time is exponential with mean rate 10 per hour, how many counters should be provided at the terminal under each of the following conditions?

    a. The expected waiting time in the system per customer does not exceed 15 minutes.

    b. The expected number of customers in the system is at most 10.

    c. The probability that all counters are idle does not exceed .11.

13. In a bank, customers arrive in a Poisson stream with mean 36 per hour. The service time per customer is exponential with mean .035 hour. Assuming that the system can accommodate at most 30 customers at a time, how many tellers should be provided under each of the following conditions.

    a. The probability of having more than 3 customers waiting is less than .2.

    b. The expected number in the system does not exceed 3.

14. In a parking lot there are 10 parking spaces only. Cars arrive according to a Poisson distribution with mean 10 per hour. The parking time is exponentially distributed with mean 10 minutes. Find the following.

    a. The expected number of empty parking spaces.

    b. The probability that an arriving car will not find a parking space.

    c. The effective arrival rate of the system.

15. In a self-service facility arrivals occur according to a Poisson distribution with mean 50 per hour. Service time per customer is exponentially distributed with mean 5 minutes.

    a. Find the expected number of customers in service.

    b. What is the percentage of time the facility is idle?

16. 10 machines are being attended by a single overhead crane. When a machine finishes its load, the overhead crane is called to unload the machine and to provide it with a new load from an adjacent storage area. The machine time per load is assumed to be exponential with mean 30 minutes. The time from the moment the crane moves to service a machine until a new load is installed is also exponential with mean 10 minutes.

    a. Find the percentage of time the crane is idle.

    b. What is the expected number of machines waiting for crane service.

17. 2 repairmen are attending five machines in a workshop. Each machine breaks down according to a Poisson distribution with mean 3 per hour. The repair time per machine is exponential with mean 15 minutes.

    a. Find the probability that the two repairmen are idle. That one repairmen is idle.

    b. What is the expected number of idle machines not being serviced?