Math/Mgmt 3502 (Ng/Fall 2007)
Handout - on continuation to CPM-PERT
November 13 - 21, 2007.

3.2 Finding Critical Paths

Here are the main steps in determining the critical path(s).

1. **The forward pass**. This is done from the "start activity" to the "end activity".
   Calculate the *earliest start time*, ES_v, and the *earliest finish time*, EF_v, for all activities v.

Step 1. For all activities, v, without immediate predecessors other than the "start activity",

$$ES_v = 0$$

$$EF_v = ES_v + t_v$$

where t_v is the duration of activity v.

Step 2. For all activities, w, with immediate predecessors,

$$ES_w = max\{EF_v \ : \ v \text{ is a predecessor of } w\}$$

$$EF_w = ES_w + t_w$$

2. **The backward pass**. This is done from the "end activity" to the "start activity".
   Calculate the *latest start time*, LS_v, and the *latest finish time*, LF_v, for all activities v.

Step 1. For all activities, v, who are the immediate predecessors of the "end activity",

$$LF_v = EF_v$$

$$LS_v = LF_v - t_v$$

where t_v is the duration of activity v.

Step 2. For all activities, w, who are immediate predecessors of activities other than the "end activity",

$$LF_w = min\{LS_v \ : \ v \text{ is a successor of } w\}$$

$$LS_w = LF_w - t_w$$

3. For each activity v, the slack time associated with v, denoted ST_v, is $LS_v - ES_v$ or $LF_v - EF_v$.

4. An activity is said to be *critical* if a delay in its start will cause a delay in the completion date of the entire project. In other words, these critical activities have zero slack times.
   Non-critical activities have non-zero *slack* times.

5. A *directed path* from node u to node v is a non-empty sequence of alternating nodes and arcs, namely, $u, (u, w_1), w_1, (w_1, w_2), w_2, \ldots, w_n, (w_n, v), v$.

6. A **critical path** is a directed path from the "start activity" to the "end activity" whose intermediate nodes consists of ONLY critical activities.
   Note: in some cases there could be alternative critical paths, but the **length** of the critical path is **unique**.
   (The length of the critical path gives the optimal time of completion for the project, provided the critical activities start and finish when they are supposed to).

Example 3.3 Consider the network representation given in Figure 1. Determine

1. the ES, EF, LS and LF times for all activities.
2. the slack time for all activities
3. the critical activities.
4. the critical path(s).
5. the length of the critical path.

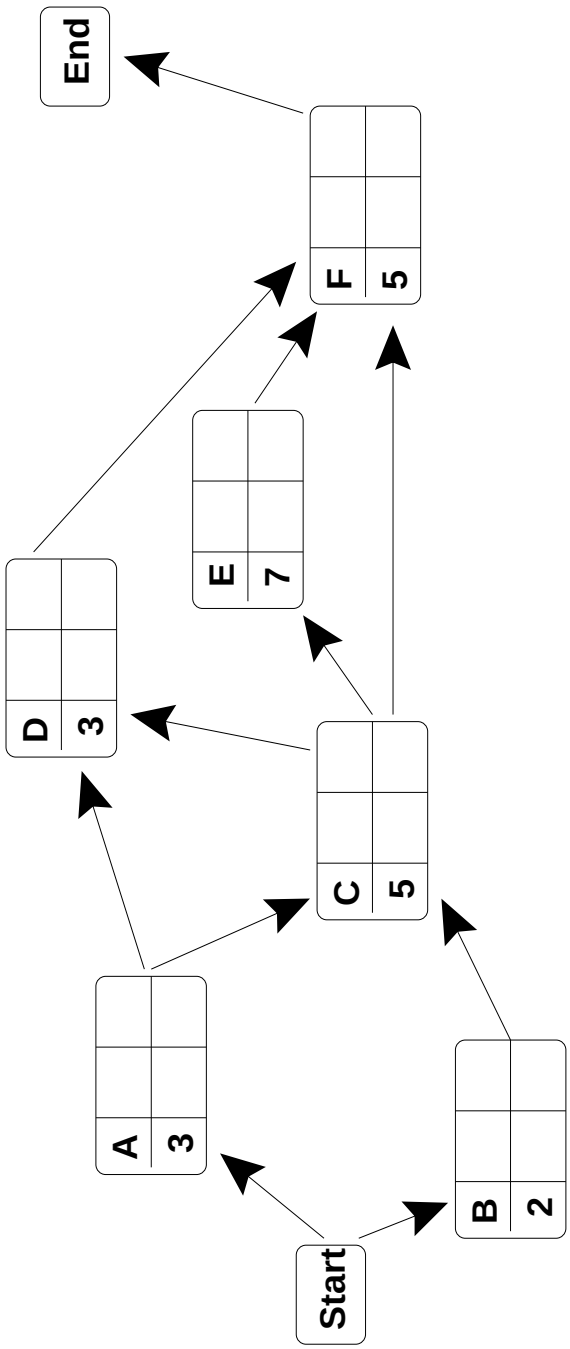

Figure 1 : Network representation for Example 3.3