

Math/Mgmt 3502 (Ng/Fall 2007)
 Handout 2 - on Introduction to CPM-PERT
 November 13 - 21, 2007.

3. Project Scheduling and Control by CPM-PERT

3.0 Motivation

The successful and intelligent management of large-scale projects requires careful and efficient planning, scheduling, and coordinating of numerous interrelated activities.

Example 3.1 Suppose we consider the scheduling of tasks involved in building *Peh's* house on a piece of existing land located at an empty lot over on the west side of *Pomme de Terre* in Morris, Minnesota. We would like to determine in what sequence the tasks should be performed in order to minimize the total time required to execute the project. All we really know is how long it takes to carry out each task and which tasks must be completed before commencing any particular task. (In fact, we only need to know the tasks that *immediately* precede a particular task, since completion of all *earlier* tasks will be implied by this information). The tasks that need to be performed in building her house, their immediate predecessors, and an estimate of their duration are given in the table below.

<i>Activity Code</i>	<i>Task</i>	<i>Immediate predecessor(s)</i>	<i>Duration (days)</i>
A	Clear site	-	1
B	Bring utilities to site	-	2
C	Excavate	A	1
D	Pour foundation	C	2
E	Outside plumbing	B,C	6
F	Frame house	D	10
G	Electric wiring	F	3
H	Lay floor	G	1
I	Lay roof	F	1
J	Inside plumbing	E,H	5
K	Shingling	I	2
L	Outside sheathing insulation	F,J	1
M	Install windows and outside doors	F	2
N	Brick work	L,M	4
O	Insulate walls and ceiling	G,J	2
P	Cover walls and ceiling	O	2
Q	Insulate roof	I,P	1
R	Finish interior	P	7
S	Finish exterior	I,N	7
T	Landscape	S	3

Although this problem could be attacked by ways of Integer Programming, there is a more efficient procedure based on the techniques of network optimization on *directed graphs* to solve such scheduling problems. Such techniques are called the **critical path method** a.k.a. (CPM), and the **project evaluation and review technique** a.k.a. (PERT).

3.1 Terminologies and Network Representations

- A *project* is a set of interrelated activities that must be executed in a certain order before the entire task can be completed.
- An *activity* is one of the tasks required by the project. Usually, an activity requires time and resources for its completion. An activity is represented by a node or a vertex in the network representation.
- The *network diagram representation* (i.e. the directed graph $G = (V, A)$) of a project is constructed in the following manner. Each vertex or node in V corresponds to an activity. For every activity $v \in V$ i.e., within each node $v \in V$, there are 6 attributes that describes the *identity of activity* (v), the *duration of activity* v denoted (t_v), the *earliest start time of activity* v (ES_v), the *earliest finish time of activity* v (EF_v), the *latest start time of activity* v (LS_v), the *latest finish time of activity* v (LF_v). See **Figure 1** for an example.

v	ES_v	EF_v
t_v	LS_v	LF_v

Figure 1 : Lay-out of an activity node, v

- An *immediate predecessor* relationship is represented by an arc $(v, w) \in A$ where activity (or node) v is an immediate predecessor of activity (or node) w . If such an arc $(v, w) \in A$, we can also say that activity w is an *immediate successor* of activity v .

See **Figure 2** for an example where activity v is said to be an *immediate predecessor of activity* w , or activity w is said to be an *immediate successor of activity* v .

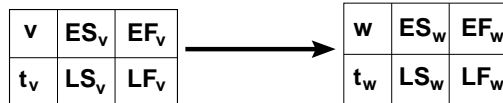


Figure 2 : Lay-out of a predecessor relationship, (v, w) for activities v and w

Example 3.2 Construct the network representation, without the numerical attributes, of a project consisting of activities A, B, C, \dots, L , such that the following relationships are satisfied.

1. A, B , and C , the first activities of the project, start simultaneously.
2. A and B precede D .
3. B precedes E, F , and H .
4. F and C precede G .
5. E and H precede I and J .
6. C, D, F , and J precede K .
7. K precedes L .
8. I, G , and L are the terminal activities of the project.