MATH 4452 (Ng/Fall 2008)
Assignment 9
on Review of basic probability
due November 18, 2008.

(**Note:** The purpose of this set of assignment is for you to recall the concepts from mathematical statistics OR probability courses that you have taken; one way to do so is to actually solve a few problems that use those concepts.

Please make sure you **define** all your events or variables clearly. In your solution, please include your justifications of your work; avoid a string of numbers and equalities without telling me where they come from.

Each problem is worth 10pts.)

1. Suppose each of three persons tosses a coin. If the outcome of one of the tosses differs from the other outcomes, then the game ends. If not, then the persons start over and retoss their coins.
   (a) Assuming fair coins, what is the probability that the game will end with the first round of tosses?
   (b) If all three coins are biased and have a probability $\frac{1}{4}$ of landing heads, then what is the probability that the game will end at the first round?
2. Suppose that 5 percent of men and 0.25 percent of women are color-blind. A color-blind person is selected at random from a population pool where there are an equal number of men and women. What is the probability of this selected person being male?
3. Suppose a coin having probability 0.7 of coming up heads is tossed three times. Let X denote the number of heads that appear in the three tosses. Determine the probability mass function of X.
4. An individual claims to have extrasensory perception (ESP). As a test, a fair coin is flipped ten times, and he is asked to predict in advance the outcome. Our individual gets seven out of ten correct. What is the probability that he would have done this well if he had no ESP?
   (Explain why the relevant probability is $P\{X \geq 7\}$ and not $P\{X = 7\}$, where X is the appropriate random variable that you should define.)
5. Derive the mean and variance for the discrete random variable, X, where X is *geometric* with parameter p. Technically, p is the probability of a "success" in an experiment. In other words, the probability mass function of X, the number of trials until a "success" occurs, is
$$P\{X = i\} = p(1-p)^{i-1} \text{ for } i = 1, 2, 3, \ldots$$
6. Derive the mean and variance for the continuous random variable, X, where X is *uniform* over the interval $[a, b]$ where $a < b$. In other words, the probability density function of X is
$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$
7. Derive the mean and variance for the continuous random variable, X, where X is *exponential* with parameter λ. In other words, the probability density function of X is
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$
8. Let X be a random variable with probability density function:
$$f(x) = \begin{cases} c(1-x^2) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
   (a) What is the value of c?
   (b) What is the cumulative distribution function of X, i.e. what is the function:
$$F(x) = P\{X \leq x\} = \int_{-\infty}^{x} f(t)dt =$$
9. Handout 11 Problem 6.