MATH 4452 (Ng/Fall 2008)
Assignment 8
due Thursday November 7, 2008.

1. (10pts.) Use mathematical induction on $|V|$ to prove that

$$|V| = |E| + 1$$

for **any** tree $T = (V, E)$.

2. (30pts.) The Armed Forces of the North Atlantic Treaty Organizations (NATO) are getting ready to distribute their troops to various countries on this great earth to fight for what they think is called democracy.

There are certain bases in cities of various countries from which the UN Marines, Army, Navy, and Air Force plan to ship the arm forces; and each of these bases has a limited number of soldiers. Table 1 gives the respective base location and the number of service men and women (**in tens of thousands**) available.

Cities/Bases	London	Norfolk, VA	Brussels
Availability	35	20	15

Table 1 : Base locations

From previous experience in wars and from forecasting techniques, the logistics department of the arm forces came up with certain number of men and women needed at these countries to achieve the goals of these invasions. Table 2 summarizes these required numbers, and also the countries in this world where the aforementioned troops are planning to invade, in the name of freedom.

Countries	Bosnia	Macedonia	Yugoslavia	Somalia
Requirements	15	10	30	15

Table 2 : Troops Data

The unit costs (**in hundreds of dollars**) of shipping these human resources from all of the bases to each of the foreign countries are given in the Table 3.

	Soon-to-be invaded countries			
Bases	Bosnia	Macedonia	Yugoslavia	Somalia
London	6	8	4	9
Norfolk, VA	5	7	9	6
Brussels	4	8	10	7

Table 3 : Table of unit costs

The Secretary General of the United Nations and his logistics experts would like to minimize the total costs of distributing their troops in such a way that demands are met and at the same time supplies are also consumed.

(a) How would you model the aforementioned problem?

(b) Formulate the above problem as a model on directed graphs. **Be explicit and very clear about your formulation or model of the problem. Make sure you say what information is needed and what you want to do.**

(c) Formulate the above problem as an LP model. Find an optimal solution via TORA's Linear Programming Module; identify an optimal solution as well as the optimal value.

3. (20pts.) Now that the troops are back from invading other countries, the United Nations is starting to think about how to use the high tech electronics power that they had developed for the wars above. One application is to build a new information superhighway communications network. The purpose of this network superhighway is so that the following cities could communicate with each other either directly or via another city.

   Table 4 gives the costs (in **millions of dollars**), (including construction costs and money to bribe the countries' politicians), of installing communication lines between any two cities.

	Tokyo	Seattle	Paris	London	Brussels	New York	Sydney
Tokyo	-	1	2	4	6	5	7
Seattle		-	6	9	10	12	16
Paris			-	4	7	8	10
London				-	6	6	9
Brussels					-	3	7
New York						-	8
Sydney							-

Table 4 : Table of costs for network layout

   (a) Design a network communication layout that will enable the United Nations to achieve that application, and to do so at minimum total cost.

   (b) Using the Greedy Algorithm, exhibit a minimum cost network and identify its minimum cost.

   (c) Using TORA's network module, exhibit a minimum cost network and identify its minimum cost.

4. (20pts.) Suppose Jimmy Joe is a national salesman who travel around the country, promoting a popular southern cuisine called grits. His home base is in Vuldasto and he would like to know how he should travel around all the cities given in Table 5, visiting each city exactly once, and return to his hometown. Of course, he would like to know the cheapest way to do so; the numbers in Table 5 give the cost from city to city.

	Vuldasto	Los Angeles	Seattle	Minneapolis	New York	St. Louis
Vuldasto	-	2	4	6	5	7
Los Angeles		-	9	10	12	16
Seattle			-	7	8	10
Minneapolis				-	6	9
New York					-	7
St. Louis						-

Table 5 : Table of costs for Jimmy Joe

   (a) Formulate the above problem as a model on undirected graph.

   (b) Formulate the problem as an *ILP*. Using TORA, find an optimal solution and optimal value for Jimmy Joe. `If you solve this problem to his satisfaction, you will be rewarded a box of his infamous grits as a sample.`

5. (20pts.) Figure 1 gives a few graphs $G = (V, E)$.

   (a) Identify those which are *eulerian* and those which are not. Justify your answers; do not just say yes or no.

   (b) For those which are not eulerian in part (a.), give an eulerization of the graph.

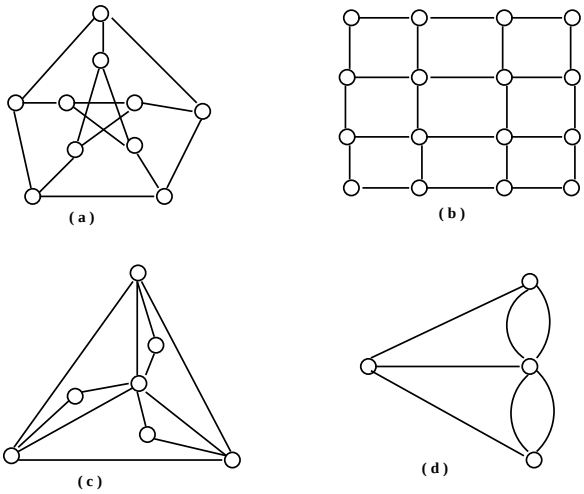

Figure 1 : Graphs, $G = (V, E)$ for *Euler* problem

6. (10pts.) A newspaper boy named Buddy must deliver papers on both sides of both Campbell Avenue and Maplewood Avenue between 63rd Street and 66th Street, and on both sides of 63rd to 66th Streets between Campbell and Maplewood Avenues. The map is given in **Figure 2**. It takes Buddy 3 minute to pedal each block plus 10 seconds for each paper he tosses on a porch. He must start and finish his paper route at the corner of 63rd and Maplewood. There are 5 houses on each side of each block.

   Buddy has hired you to help him find the best route (**trail**) for **Buddy** to deliver the newspapers. (Best meaning the fastest.) How would you advise **Buddy** to **start** solving his problem, in other words, could you tell him what kind of graph models he can use to solve his problem? (I did NOT ask you to give him an optimal solution.)

   (**Warning:** When you are giving your graph model in writing, please make sure that the problem on the graph in your description is precise and clear, in such a way that there is no ambiguity pertaining to the paper boy problem).

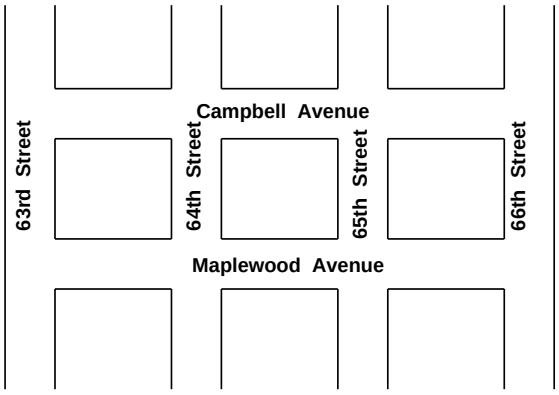

Figure 2 : Partial Map for Buddy's newspaper route