MATH 4452 (Ng/Fall 2008)
Assignment 6
due Fri October 17, 2008 in my office.

1. (20pts.) (*Regional Planning*). A few counties in the west central region of Minnesota are planning to have an agricultural consortium based on communal farming communities. The motivation behind it is to get groups of local farmers to join together and share common technical services and to coordinate their production.

   The Minnesota's Department of Agriculture decided to perform a feasibility study of having such a consortium based on just three counties to start with. They called this group the Consortium of Norwegian Counties Farmers. The overall planning is done in the Department of Agriculture in Minneapolis/St. Paul, and they are planning agricultural production for the coming year.

   The agricultural output of each county is limited by both the amount of available irrigable land and by the quantity of water allocated for irrigation by the Department of Water Works. These data are provided in Table 1.

Counties	*Usable Land* (acres)	*Water Allocation* (acre feet)
Stevens	400	600
Pope	600	800
Stearns	300	375

Table 1 : Resources Data for Consortium of Norwegian Counties Farmers

The crops suited for this region include corn, wheat and soybean, and these are the three crops being considered for the upcoming season. These crops differ primarily in their expected net return per acre and in their consumption of water. In addition, since bureaucracy **loves** control, the Department of Agriculture has set a maximum quota for the total acreage that can be devoted to each of these crops by the Consortium of Norwegian Counties Farmers as shown in Table 2.

Crop	*Maximum Quota* (acres)	*Water Consumption* (acre feet per acre)	*Net Return* (dollars per acre)
Corn	600	3	400
Wheat	500	2	300
Soybean	325	1	100

Table 2 : Crop Data For Consortium of Norwegian Counties Farmers

These three counties belonging to the Consortium of Norwegian Counties Farmers have agreed that every county will plant the same proportion of its available irrigable land. For example, if Stevens County plants 200 of its availbale 400 acres, then Pope must plant 300 of its 600 acres, while Stearns must plant 150 of its 300 acres. However, any combination of the crops may be grown within any of the three counties.

Since the planning is done by the Department of Agriculture, this office must decide how many acres to devote to each crop at the respective counties while satisfying all the aforementioned restrictions, and to do so in such a way that the total net return to the Consortium of Norwegian Counties Farmers is maximized.

(*Your mission, should you choose to accept, is to formulate a linear programming model that will solve the Department of Agriculture's plan for the Consortium to work.*)

2. (20pts.) Bubba and Bubbette Johnson procreated eight years ago. In anticipation of the immense college expenses for Bubba Jr., they decided to start an annual investment program on the child's eighth birthday (now) that will last until the eighteenth birthday. Judging from Bubba and Bubbette's financial position over the next 10 years, the couple estimates that they will be able to invest the following amounts at the beginning of each year:

Year	1	2	3	4	5	6	7	8	9	10
Amount (\$)	2000	2000	2500	2500	3000	3500	3500	4000	4000	5000

To avoid unpleasant surprises, Bubba and Bubbette opt to invest the money very safely. The following options are open to them:

(a) Insured savings with 5.5% annual yield.

(b) Six-year government bonds that yield 6.5% annually and have a current market price equal to 0.96 of the face value.

(c) Nine-year municipal bonds yielding 7.2% annually and having a current market price equal to 1.03 of the face value.

Bubba and Bubbette would like to know how they should invest their money over the next 10 years so that their total yield at the end of the tenth year will be maximized. Formulate the couple's problem as an LP model; you are not asked to solve it.

(F.Y.I., market price of bonds is the price the investor actually pays to buy the bonds. Each bond yields simple annual interest on its face value, and its face value is surrendered at maturity date. In addition, the annual interest paid for any of the types of investments are reinvested into any of the three types of investments at any year.)

3. (15pts.) Farmer Jack wishes to determine the best selection of stock for his farm, his objective being to maximize his profit after the sale of the animals at the end of the period. The alternatives available are *Merino* sheep, *Romney* sheep, *Southdown* sheep, *Hereford* cattle and *Jersey* cattle. Jack has calculated that each Merino would require one acre of land, and would cost \$1.50 in extra feed, treatment, etc. The purchase price is \$160 and Jack estimates that the selling price at the end of the period will be \$300. The following table gives the respective data for the different types of the stock.

	Land req.	Unit cost in feed	Purchase price	Selling price
Merino	1 acre	\$1.50	\$160	\$300
Romneys	1 acre	\$1.75	\$125	\$250
Southdowns	1 acre	\$1	\$100	\$200
Herefords	4 acres	\$15	\$130	\$210
Jerseys	6 acres	\$12	\$180	\$320

The size of the farm is $1,000$ acres, and the farmer has $\$15,000$ with which to purchase and to maintain the stock.

a. (10pts.) Formulate the problem as a linear program.

b. (5pts.) It is conceivable that an optimal solution to your problem in part (a) could contain fractional components. What constraints do we have to add to the LP in part (a) for the model to be more realistic? (If correct, this new formulation is called an *integer* linear program).

4. (20pts.) The school board of the Smorris Area Schools has made the decision to close down the Crack Road Junior High middle school (sixth, seventh, and eighth grades) at the end of this school year and reassign all of next year's middle school students to the three remaining middle schools, namely, Smorris Junior High, Northside Junior High, and Gundown Road Junior High.

   The school district provides bussing for all middle school students who must travel more than approximately a mile, so the school board needs a plan for reassigning the students so that their total bussing cost is minimized. The cost per student of bussing from each of the six residential areas of the city to each of the schools is shown in Table 3 along with other basic data for next year. (0 *means no bussing is needed, and* − *means that particular assignment is not possible.*)

					Bussing Cost Per student		
Area	*Number of Students*	*% 6th Grade*	*% 7th Grade*	*% 8th Grade*	*Smorris Jr. High*	*Northside Jr. High*	*Gundown Jr. High*
1	450	32	38	30	3	0	7
2	600	37	28	35	0	4	5
3	550	31	32	37	6	3	2
4	350	28	39	33	2	5	–
5	500	39	34	27	0	–	0
6	450	33	29	38	5	3	4
			School capacity:		900	1,100	1,000

Table 3 : Table of Data for Bussing Middle Grade Students

   The school board also has imposed the restriction that each grade must constitute between 30 and 35 percent of each school's population. Table 3 shows the percentage of each area's middle school population for next year that falls into each of the three grades. The school attendance zone boundaries can be drawn so as to split any given area among more than one school, but assume that the percentages shown in Table 3 will continue to hold for any partial assignment of an area to a school.

   a. (15pts.) Formulate a linear programming model for determining how many students should be assigned from each area to each of the three schools so that the frugal City of Smorris could spend the least amount of money on bussing middle grade students.

   b. (5pts.) It is conceivable that an optimal solution to your problem in part (a) could contain fractional components. What constraints do we have to add to the LP in part (a) to guarantee that the solutions will be more realistic? (If correct, this new formulation is called an *integer* linear program).

5. (15pts) (**A Transportation Problem**).

   The Brazilian Coffee Company (a.k.a. BCC) processes raw coffee beans into usable coffee beans at 3 plants. The usable coffee beans are then shipped every week to 7 warehouses in major cities for retail, distribution and exporting purposes. The unit shipping cost from plant i to warehouse j is c_{ij} for $i = 1, 2, 3$ and for $j = 1, 2, 3, 4, 5, 6, 7$. The production capacity of plant i is at most a_i for $i = 1, 2, 3$; and the demand at warehouse j is at least b_j for $j = 1, 2, 3, 4, 5, 6, 7$. Formulate an LP to find the production-shipping pattern that minimizes the overall shipping cost.