MATH 4452 (Ng/Spring 2012)
Assignment 1
due January 26, 2012.

1. (10pts.) Refer to Example 2 in Section 1.1 of your notes on Population Growth Model with Growth Limit. Show that the rate of growth of population is maximum when

$$P(t) = \frac{M}{2}$$

where M is the limiting population.

2. (10pts.) Refer to the same example on Population Growth Model with Growth Limit. Show that the time $\tilde{t}$ such that

$$P(\tilde{t}) = \frac{M}{2}$$

is

$$\tilde{t} = -\frac{1}{k} \ln \frac{P_0}{M - P_0}$$

3. (30pts.) (Learning Curve). Psychologists believe that when a person is asked to recall a set of facts, the rate at which the number of facts are recalled is directly proportional to the difference between M, the total number of relevant facts in a person's memory, and the number of facts that the person recalls after t minutes.

   (a) Derive a model to determine the number of facts that Bubba can recall after t minutes, given that the total number of facts that Bubba can hold in his brain is 200.

   (b) Sketch or plot (using *Mathematica*), the graph of the number of facts Bubba can recall as a function of time in minutes.

   (c) What happens to the graph as t increases without bound, i.e. find

   $$\lim_{t \to \infty} P(t)$$

   where $P(t)$ denotes the number of facts Bubba can recall after t minutes.

4. (50pts.) (Spread of a rumor or an epidemic). The total population of an island-resort called Diseaseland is N. A small group of them have been found to be infected with *influenza*. The National Institute of Health, a.k.a. NIH, have reasons to believe that the rate at which influenza spreads through the community is jointly proportional to the number of people in the community who have caught the disease and those in the community who have yet to catch it.

   (a) Write down the rate equation of the aforementioned problem. Graph this rate equation, i.e. graph the rate versus the number of people who have caught influenza. (**Make sure you define all the variable you use**).

   (b) Derive a mathematical model that will describe the number of people in the community who have caught influenza as a function of time.

   Identify this model, i.e. does it correspond to something that was done in class?

   (c) Graph the function you have in part (b.)

   (d) At what time is the disease spreading at the fastest rate?

   (e) According to your model, how many people will eventually be infected?

   (f) How will your answer to part (c) be different in the following two scenarios. If the initial number of people with influenza is less than half the total population; or if the initial number is more than half the total population.

   (g) **Time to test your model**. Suppose the following data are collected at various times on Diseaseland which has a total of 4,000 people. Do they support your model, i.e., is the assumption made by the NIH reasonable?

t days	1	4	7
number of people infected	436	1156	2298