

<p>MATH 4452 (Ng/Spring 2012)</p>

<p>Handout 5</p>

<p>Lotka-Volterra Equations (continuous Predator-Prey model) - CDS</p>

<p>For class on February 7-14, 2012.</p>

1. Lotka-Volterra Equations for *Predator-Prey Models*.

This is an extension of population growth with limits in Section 1.2 with the logistics growth. Now, we would like to model the population growth of two populations, one of which feeds on the other. That is why the rate equation is called a *coupled system*.

For instance, the two populations could be sharks and tuna fish. The sharks will be the *parasite* or *predator* while the tuna fish will be *host* or *prey*.

The rate of growth of tuna fish (host) is proportional to the amount of tuna (host) available but at the same time an increase in the population of sharks (parasite) will decrease the population of tuna.

The rate of growth of sharks (parasite) depends on the availability of sharks and also the availability of tuna fish (host) for feeding purposes. If the population of tuna (host) is large, then the population of sharks will increase, while if the population of tuna is small then the population of sharks will decrease.

Let H or $H(t)$ be the population of tuna fish (host) at time t ; let P or $P(t)$ be the population of sharks (parasite) at time t .

- Set up the rate equations for this coupled system.
- What are the *equilibrium points*, (points at which the rates of growth are zero)?
- Can an closed form solution be obtained for both $P(t)$ and $H(t)$? Can an analytical solution be obtained implicitly for the relationship between P and H ? Identify them.
- Identify any relationship, graphically, between the population of tuna fish and sharks.

2. (Crazy Lotka-Volterra model.)

Rhett and Scarlett are two lovers who share and thrive on a *sadistic relationship*, which is typical of immature people.

The way their relationship evolves as time goes by can be described in the following paragraph.

The more Rhett loves Scarlett, the more she begins to dislike him. On the other hand, when Rhett's love for Scarlett begins to taper off, the more her affection for him begins to grow. In other words, if the amount of Rhett's love is higher then the rate of growth of Scarlett's love is smaller.

For Rhett's part, his love for Scarlett grows when she loves him and dissipates when she dislikes him. In other words, if the amount of Scarlett's love is higher then the rate of growth of Rhett's love is also higher.

Let $R(t)$ be (the amount of) Rhett's love or hate for Scarlett at any time t ; and let $S(t)$ be (the amount of) Scarlett's love or hate for Rhett at any time t . (*For either function, a positive value means love and a negative value means hate*).

As a simple case, let us assume that *the rate of change of Rhett's love or hate* has a linear relationship with *the amount of Scarlett's love or hate*; and the former is zero when the latter is zero. And similarly, we assume that *the rate of change of Scarlett's love or hate* also varies linearly with *the amount of Rhett's love or hate*; and the former is zero when the latter is zero.

- Based on the above description, set up the equation that describes the *rate of change of Rhett's love or hate for Scarlett* with respect to *time*, at any time t . (**Specify the condition on the constant of proportionality.**)
- Set up the equation that describes the *rate of change of Scarlett's love or hate for Rhett* with respect to *time*, at any time t . (**Specify the condition on the constant of proportionality.**)
- Based on the two coupling differential equations you obtained in parts (a) and (b), set up the equation that describes the *rate of change of Scarlett's love or hate with respect to Rhett's love or hate*. (**Hint: Use the Chain Rule**).
- Based on your equation in part (c), solve for a direct relationship between S , (Scarlett's love/hate), in terms of R , (Rhett's love/hate). (In other words, come up with an equation model that will describe the relationship between S and R .)
- Based on your answer to part (d), sketch the graph of S versus R . Explain, clearly, why Scarlett and Rhett's relationship is a *never-ending cycle of love and hate*.