MATH 3411 (Ng/Fall 2010)
Handout 9
Algorithms for Shortest Directed Path (SP) Problem
for class on **November 16, 2010.**

1 Dijkstra's Algorithm (DA)

Given or Input:

(i) A directed graph $G = (V, A)$; a *distinguished vertex*, $s \in V$,

(ii) **non-negative** real valued costs $c_{ij} \geq 0$ for all $(i, j) \in A$.

Want or Output:

a shortest directed path from s to t and its length.

NOTATIONS:

Step 0 (**Initialization**)

(i) Set

$$\nu(i) \longleftarrow \begin{cases} c_{s,i} & \text{if arc } (s,i) \in A \\ 0 & \text{if } i = s \\ +\infty & \text{otherwise} \end{cases}$$

(ii) Make source vertex, s, as *permanently labelled*, while others are temporarily labelled.

Main Step If all vertices are permanently labelled, STOP. Otherwise, choose vertex $\hat{i}$, a temporarily labelled vertex whose $\nu(\hat{i})$ value is the **smallest** among *all* temporarily labelled vertices, i.e. vertex $\hat{i}$ is a temporarily labelled vertex such that:

$$\nu(\hat{i}) = \min\{\nu(i) \,:\, \text{vertex } i \text{ is temporarily labelled vertex}\}$$

Make $\hat{i}$ *permanently labelled*

Go to Update Step

Update Step If all vertices are permanently labelled, STOP. Otherwise, update only the temporarily labelled vertices, i, in the following way:

$$\nu(i) \longleftarrow \min\{\nu(i)\,,\, \nu(\hat{i}) + c_{\hat{i},i}\}$$

Repeat **Main Step**

2 Floyd-Warshall's Algorithm (FW)

Given or Input:

(i) A directed graph $G = (V, A)$;
(ii) real valued costs c_{ij} for all $(i, j) \in A$,
(iii) with **NO negative weight directed cycles**

Want or Output:

shortest directed paths from **all** vertices to **all** vertices, and their lengths.

NOTATIONS:

Step 0 (**Initialization**)

(i) Set

$$\nu^0(i,j) \longleftarrow \begin{cases} 0 & \text{if } i = j \\ c_{i,j} & \text{if arc } (i,j) \in A \\ +\infty & \text{if arc } (i,j) \notin A \end{cases}$$

Main Step (**Triple "*For*" Loops**)

For $m = 1, 2, 3, \ldots, |\ V\ |$, do:

$$\begin{cases} \text{For all } i \text{ and } j,\ i \neq m\ ,\ j \neq m, & \text{update:} \\ \nu^m(i,j) \longleftarrow & \min\{\ \nu^{m-1}(i,j)\ ,\ \nu^{m-1}(i,m) + \nu^{m-1}(m,j)\ \} \\ \text{If } \nu^m(i,i) < 0\,, & \text{then STOP; } \exists \text{ a negative weight directed cycle} \end{cases}$$