1. Reviews for Exams 1, 2, & 3

2. Applications

Given a description of a real-world problem, you should know how to formulate it as a combinatorial optimization problem, and know which correct optimization problem to apply in order to solve the real-world problem.

3. Shortest Directed Path (SP)

   (a) You should know statement of the problem for an (SP).

   (b) You should know how to model an application problem as one of shortest path, i.e. know what is given and what you want to find.

   (c) You should know the required restrictions for the Dijkstra and the Floyd-Warshall’s algorithm to work.

   (d) You should know how to apply the Dijkstra and the Floyd-Warshall’s algorithm to find the shortest directed path(s). At the end of the algorithm, you should know how to identify the optimal solutions (shortest directed paths) and the optimal value (the length of the shortest directed paths.)

      (Note: there could be multiple optimal solutions, but there can only be a unique optimal value, if one exists.)

   (e) You should know how to apply $F - W$’s algorithm to finding a negative weight directed cycle in a directed graph, if one exists.

   (f) You should know the rationale behind why Dijkstra and the Floyd-Warshall’s algorithms work.

4. Maximum $s-t$ Flow Problem (MF) and Minimum Capacity $s-t$ Cut Problem (MC)

   (a) You should know statement of the problem for (MF), i.e. what information is given, and what you you are looking for.

   (b) You should know statement of the problem for (MC), i.e. what information is given, and what you you are looking for.

   (c) You should know how to model an application problem as one of (MF) or (MC), i.e. know what is given and what you want to find, and how those two sets of information fit into an instance of (MF) or an instance of (MC).

   (d) You should know how to use the Ford-Fulkerson’s (FF) algorithm to find a maximum feasible $s - t$ flow.

      There is a big difference between a maximum feasible $s - t$ flow and the value of a maximum $s - t$ flow.

   (e) Be familiar with all the weak duality type lemma, the strong duality type lemma, and the max-flow min-cut theorem.
(f) You should be familiar with all other results relating the two different combinatorial optimization problems, (MF) and (MC), in particular, how (MC) is solved once (MF) is solved.

(g) NEVER solve (MC) via trials of several sets, $S$, or by total enumeration.
Reason: your final exam is NOT short, and you need to optimize your time.

(h) You should be familiar with the terms:
   i. an s-t flow, i.e., an s-t flow is an assignment of values on the arcs of the given directed graph,
   ii. a feasible s-t flow, i.e. a feasible s-t flow is an s-t flow that satisfies two conditions, namely, the flow on each arc must be non-negative and cannot exceed its capacity, and, total flow into any vertex $k \in V \setminus \{s, t\}$ must be the same as the total flow out of vertex $k$.
   iii. the value of a feasible s-t flow, i.e. the value of a feasible s-t flow is the net flow out of the source, $s$, or the net flow into the sink, $t$, or the net flow across the arcs of any s-t cut.
   iv. Given a set of vertices $S$ such that $s \in S$ and $t \notin S$, an s-t cut defined by $S$ denoted $\delta(S)$, is a set of arcs (not vertices) that go from vertices in $S$ to vertices NOT in $S$ OR from vertices not in $S$ to vertices in $S$.
Technically, 
\[
\delta(S) = \{(i, j) \in A : (i \in S \land j \notin S) \lor (i \notin S \land j \in S)\}
\]
   v. Given an s-t cut, the capacity of the s-t cut, denoted $\text{cap}($\(\delta(S)\)) is the sum of all the capacities of the forward arcs in $\delta(S)$. (Forward arcs in an s-t cut are arcs $(i, j)$ in the s-t cut such that $i \in S \land j \notin S$.)
For instance, from the directed graph $G = (V, A)$ in Figure 1, with capacities $u_{ij} \geq 0$, and with $s = 1$ and $t = 2$, the following are examples of s-t cuts defined by vertex sets $\tilde{S}$.
   • Let $\tilde{S} = \{1, 6\}$. Then, the s-t cut defined by $\tilde{S}$, $\delta(\tilde{S})$ and its capacity, $\text{cap}(\delta(\tilde{S}))$ are, respectively, 
   \[
   \delta(\tilde{S}) = \{(1, 3), (1, 4), (5, 1), (1, 5), (4, 6), (6, 5), (6, 4), (6, 2)\}
   \]
   and
   \[
   \text{cap}(\delta(\tilde{S})) = u_{13} + u_{14} + u_{15} + u_{65} + u_{64} + u_{62} = 6 + 4 + 9 + 5 + 16 + 11 = 51
   \]
   • Let $\tilde{S} = \{1, 3, 4\}$. Then, the s-t cut defined by $\tilde{S}$, $\delta(\tilde{S})$ and its capacity, $\text{cap}(\delta(\tilde{S}))$ are, respectively, 
   \[
   \delta(\tilde{S}) = \{(4, 2), (6, 4), (5, 4), (4, 6), (1, 5), (5, 1), (1, 6)\}
   \]
   and
   \[
   \text{cap}(\delta(\tilde{S})) = u_{42} + u_{46} + u_{15} + u_{16} = 7 + 11 + 9 + 10 = 37
   \]
Note: DO NOT FORGET THAT A CUT INCLUDES ALL FORWARD AND REVERSE ARCS
Figure 1: Graph for examples of $s-t$ Cuts