Math 3411 (Ng/Fall 2011)
Assignment for max flow-min cut
Due December 6, 2011.

1. A company is dividing its departments between two buildings. Maintenance units 1 and 2 (vertices 1 and 2) will stay in their respective buildings, but all other units could be in any of the two buildings. The graph in Figure 1 shows the various departments (vertices) and the traffic levels (numbers on the edges) between the departments. Our interest is in finding an allocation of departments to buildings that minimizes total between-building traffic.

   (a) (10pts.) Suppose we wish to calculate a maximum flow in the aforementioned graph from 1 to 2 using the traffic levels t_{ij} as the capacities. Since Max-Flow is a problem based on directed graphs, we need to do some transformation.

   We could do so by replacing each edge (i, j) of the undirected graph by arcs (i, j) and (j, i), both with capacity t_{ij}. Make this conversion to a **directed graph** $G = (V, A)$; **clearly, show** G**, and indicate all the capacities on all the arcs of** G**.**

   (b) (25pts.) Based on the given network (directed graph), the capacities and the appropriate source and sink vertices in part (a), compute the maximum-flow from vertex 1 to vertex 2, **using Ford-Fulkerson's Algorithm**. Use as initial flow, a flow of 2 units along the directed path $1 \to 3 \to 5 \to 6 \to 2$, and zero units on other arcs.

   Clearly, show updated feasible $s - t$ **flow, its relative residual graph** $\hat{G}$**, and an** $s - t$ **augmenting chain in** G**, at every iteration.**

   Identify the optimal solution (optimal flows on the arcs) and the optimal value (max flow value).

   (c) (7pts.) At the end of the Ford-Fulkerson's algorithm in part (b), what is S, the set of vertices that are reachable from the source s in $\hat{G}$?

   (d) (10pts.) Based on your set S in part (c), find an $s \to t$ cut, in the directed graph you created, whose capacity is of the same value as the value of the maximum flow you obtained in part (b).

   (**You must identify what the cut is - remember that an s-t cut is a set of arcs that satisfies certain conditions. And you must specify its capacity and say how you get it.**)

   (e) (8pts.) (FYI: This is a mathematical modeling question.)

   Explain, **clearly**, how the answer to our department location problem can be obtained from the results of your maximum-flow calculations.

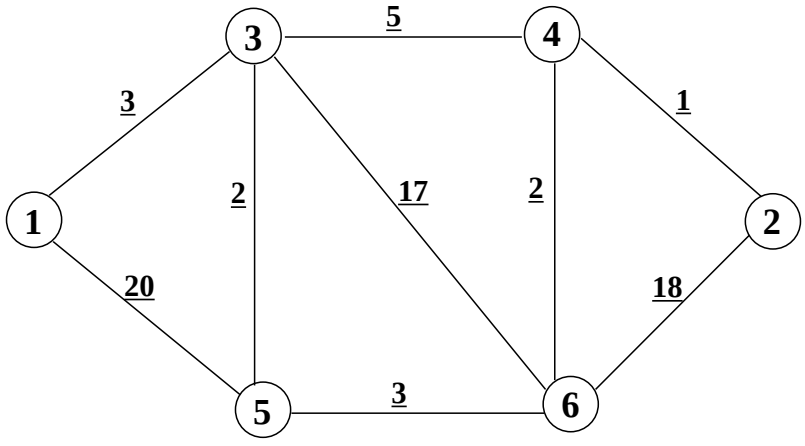

Figure 1 : Graph for Max Flow-Min Cut problem