MATH 3401 (Ng/Spring 2011)
Assignment 3
Due February 9, 2011.

Note: For problems that require the simplex method, use the dictionary format.

1. (10pts). Use the *Simplex Algorithm* to show **and** to **justify clearly** why the following (LP) is unbounded. (Also, explain in words, why you are convinced that the problem is unbounded).

$$\begin{aligned} \text{Maximize } z &= 3x_1 + x_2 \\ \text{s.t.} & \\ x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 &= 2 \quad (1) \\ x_1 + \frac{1}{2}x_3 - \frac{1}{2}x_4 &= 1 \quad (2) \\ -\frac{3}{2}x_3 + \frac{1}{2}x_4 &\leq 2 \quad (3) \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

2. (20pts). Consider the following (LP) :

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 5x_2 - 4x_3 \\ \text{s.t.} & \\ 3x_1 + 4x_2 + 2x_3 &\leq 16 \quad (1) \\ 4x_1 + 6x_2 + 2x_3 &\leq 10 \quad (2) \\ 12x_1 + 3x_2 + 6x_3 &\leq 33 \quad (3) \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

   a. Find an initial basic feasible solution with slack variables as the basic variables.
   b. Use the primal simplex method to improve the solution in part (a.). (One step).
   c. Identify the new basic and non-basic variables after part (b.)
   d. Use Primal Simplex method to solve the problem.

3. (20pts). Use the **Big-M** method to solve the following (LP)'s.

   a.

$$\begin{aligned} \text{Maximize } z &= 4x_1 - 5x_2 - 3x_3 \\ \text{s.t.} & \\ x_1 + x_2 + x_3 + x_4 &= 11 \quad (1) \\ x_1 + x_2 &\geq 1 \quad (2) \\ 2x_1 + 3x_2 + x_3 &\geq 20 \quad (3) \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

b.

$$\begin{aligned} \text{Maximize } z &= x_1 - 2x_2 - 4x_3 + 2x_4 \\ \text{s.t.} & \\ 3x_1 + 2x_2 - x_3 - 2x_4 &\leq -3 \quad (1) \\ -x_1 + x_2 - 2x_3 - x_4 &\geq 10 \quad (2) \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

4. (20pts). Solve the following (LP)'s by the **Two-Phase** method.

a.

$$\begin{aligned} \text{Minimize } z &= -2x_1 + 2x_2 + x_3 + x_4 \\ \text{s.t.} & \\ x_1 + 2x_2 + x_3 + x_4 &\leq 2 \quad (1) \\ x_1 - x_2 + x_3 + 5x_4 + x_5 &\geq 4 \quad (2) \\ 2x_1 - x_2 + x_3 &\geq 2 \quad (3) \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

b.

$$\begin{aligned} \text{Maximize } z &= x_1 - 2x_2 - 4x_3 + 2x_4 \\ \text{s.t.} & \\ 3x_1 + 2x_2 - x_3 - 2x_4 &\leq -3 \quad (1) \\ -x_1 + x_2 - 2x_3 - x_4 &\geq 10 \quad (2) \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$