MATH 2111 (Ng/Spring 2011)
Review for our Comprehensive Final Examination
Final Exam is on Mon May 9 at 11am till 1pm
for class on May 6, 2011.

1. **Reviews for Exams** 1**,** 2**, and** 3**.** Also, review your notes, and try to rework the problems/proofs I did and the problems/proofs you did.

2. **Linear Transformation**

   (a) A function, L from a vector space V to a vector space W, i.e. $L : V \longrightarrow W$, is a **linear transformation** if two conditions hold. What are these two conditions?

   (b) Given any function L from a vector space V to another vector space W, you should be able to prove whether L is a linear transformation or not.

   (c) You should be familiar with the basic results about linear transformations in 6.1, abd be able to apply these results to solve problems.

   (d) Given a linear transformation $L : V \longrightarrow W$, you should know how to show if L is *one-to-one* or *onto*.

   (e) You should know how to show if a linear transformation is **invertible** or not, i.e. L is invertible if and only if L is one-to-one and onto.

   (f) You should know the definition of the *kernel*(L) and the range(L).

   $$ker(L) = \{\vec{v} \in V : L(\vec{v}) = \mathbf{0}_W\}$$

   where $\mathbf{0}_W$ is the zero vector in W, and

   $$range(L) = \{\vec{w} \in W : \text{ there exists } \vec{v} \in V \text{ where} L(\vec{v}) = \vec{w}\}$$

   (g) You should know how to show that the kernel(L) is a subspace of V and the range(L) is a subspace of W.

   (h) Given a linear transformation $L : V \longrightarrow W$, you should know how to find the kernel(L) and the range(L), i.e. find a basis for the kernel(L) and find a basis for the range(L).

   (i) You should be familiar with the proofs of, and be able to apply the results on linear transformations, kernel(L), range(L), the dimension of the kernel(L), and the dimension of the range(L).

   (j) Given a linear transformation $L : V \longrightarrow W$, and given that S and T are bases for V and W, respectively, you should know how to find the **matrix representation of** L **with respect to** S **and** T.

   (k) You should know how to use a matrix representation of a linear transformation with respect to some ordered bases to solve problems.

3. **Eigenvalues & Eigenvectors**

   (a) You should know the definitions of an *eigenvalue*, λ, of a linear operator L on a vector space V and an *eigenvector*, $\vec{X}$, of L *associated with eigenvalue* λ.

   (b) Given that $L : V \longrightarrow V$, is a **linear operator** with some ordered basis, $S = \{\vec{v_1}, \vec{v_2}, \vec{v_3}, \ldots, \vec{v_k}\}$, and given the matrix representation A of L, you should know the connections between eigenvalues/eigenvectors of L and A, i.e.

   $$L(\vec{X}) = \lambda \vec{X} \text{ if and only if } A[\vec{X}]_S = \lambda[\vec{X}]_S$$

   where $[\vec{X}]_S$ is the coordinate vector of $\vec{X}$ with respect to the basis S (from 4.8).

   (c) You should know the definitions of an *eigenvalue*, λ, of an $n \times n$ matrix A and an *eigenvector*, $\vec{X}$, of A *associated with eigenvalue* λ.

   (d) You should know how to find the *characteristic polynomial* of an $n \times n$ matrix, A.

   (e) You should know how to find all real *eigenvalues* of an $n \times n$ matrix, A.

   (f) You should know how to find the set of *eigenvectors associated with each eigenvalue* of an $n \times n$ matrix, A.

   (g) Given an eigenvalue λ of A or of L, you should know how to mathematically describe the *eigenspace* of A or of L associated with λ.

   (h) You should also know why the eigenspace is a subspace of $\Re^n$ or V, and know how to find a basis for a eigenspace.

   (i) You should be familiar with the results on eigenvalues and eigenvectors that we learned in class or in your assignments; and you should know how to apply these results to solve given problems. In particular,

      i. What is the computational advantage of knowing eigenvalues, λ, of A and the associated eigenvectors, i.e. how we do evaluate $A^k(\vec{X})$ or $L^k(\vec{X})$ without having to multiply A by itself k times or applying the linear operator L k times?

      ii. Why are the eigenvalues of similar matrices the same?

      iii. The *Cayley-Hamilton Theorem* and how would you apply it?

      iv. How to determine if a matrix is diagonalizable, and why are diagonal matrices useful?