MATH 2101 (Ng/Fall 2010)
Review for Final Examination
on Tuesday December 14, 2010 from 8:30am till 10:30am.
(*for class on Dec 10*)

1. **Reviews for Exams 1,2,&3**

2. **Triple Integrals**

   (a) You should know how to evaluate the *Riemann sum* of $f(x,y,z)$ over a box, $B = \{(x,y) : a \le x \le b,\ c \le y \le d,\ r \le z \le s\}$ given by a certain number of subintervals in x, y and z, and certain choice of $(x^*_{ijk}, y^*_{ijk}, z^*_{ijk})$.

   (b) Given the *triple integral of f over a solid $E \subseteq \mathcal{R}^3$*, i.e. given

$$\int\int_E\int f(x,y,z)\ dV$$

   you should be able to **set up** its equivalent **iterated integral** such as:

$$\int_a^b \int_{y=g(x)}^{y=h(x)} \int_{z=p(x,y)}^{z=q(x,y)} f(x,y,z)\,dz\,dy\,dx \qquad \text{or} \qquad \int_c^d \int_{x=r(y)}^{x=s(y)} \int_{z=p(x,y)}^{z=q(x,y)} f(x,y,z)\,dz\,dx\,dy$$

   or

$$\int_a^b \int_{z=g(x)}^{z=h(x)} \int_{y=p(x,z)}^{y=q(x,z)} f(x,y,z)\,dy\,dz\,dx \qquad \text{or} \qquad \int_c^d \int_{x=r(z)}^{x=s(z)} \int_{y=p(x,z)}^{y=q(x,z)} f(x,y,z)\,dy\,dx\,dz$$

   or

$$\int_a^b \int_{z=g(y)}^{z=h(y)} \int_{x=p(y,z)}^{x=q(y,z)} f(x,y,z)\,dx\,dz\,dy \qquad \text{or} \qquad \int_c^d \int_{y=r(z)}^{y=s(z)} \int_{x=p(y,z)}^{x=q(y,z)} f(x,y,z)\,dy\,dx\,dz$$

   where g, h, r, s are real-valued functions of one variable, and q, p are real-valued functions of two variables. And, you should also be able to evaluate the iterated integrals.

   (c) **Coordinate Syatems: Rectangular, Cylindrical, Spherical**.
   You should know how to convert points in 3D among the coordinate systems. In particular, convert
   $(\bar{x}, \bar{y}, \bar{z})$ to $(\bar{r}, \bar{\theta}, \bar{z})$ to $(\bar{\rho}, \bar{\theta}, \bar{\phi})$ and vice-versa.

   (d) You should be able to convert iterated triple integrals from *rectangular coordinates* to *cylindrical coordinates* and vice-versa.
   For instance,

$$\begin{aligned}\int_E\int f(x,y,z)\ dV &= \int_a^b \int_{y=g(x)}^{y=h(x)} \int_{z=p(x,y)}^{z=q(x,y)} f(x,y,z)\,dz\,dy\,dx \\ &= \int_{\theta=\alpha}^{\theta=\beta} \int_{r=F(\theta)}^{r=G(\theta)} \int_{z=p(r,\theta)}^{z=q(r,\theta)} f(r\cos(\theta), r\sin(\theta), z)\,r\,dz\,dr\,d\theta \\ &= \int_{\theta=\alpha}^{\theta=\beta} \int_{\phi=F(\theta)}^{\phi=G(\theta)} \int_{\rho=p(\phi,\theta)}^{\rho=q(\phi,\theta)} f(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))\,\rho^2\ \sin(\phi)\ d\rho\,d\phi\,d\theta\end{aligned}$$

   (e) Applications of triple integrals. You should be able to apply triple integrals to find the volume of a solid.

3. **Vector Fields and line integrals**
   (a) You should know what is a vector fields are in $\mathcal{R}^2$, in $\mathcal{R}^3$, or in $\mathcal{R}^n$.
   (b) You should know how to evaluate the line integrals of real-valued functions of multiple variables, eg
   $$\int_{\mathcal{C}} f\,ds \text{ or } \int_{\mathcal{C}} f\,dx \text{ or } \int_{\mathcal{C}} f\,dy$$
   where $\mathcal{C}$ is a curve that you must know how to parameterize.
   (c) You should know how to evaluate the line integrals of vector fields, i.e.
   $$\int_{\mathcal{C}} \vec{F} \bullet d\vec{r} = \int_a^b \vec{F}\,(\vec{r}(t)) \bullet \vec{r}\,'(t)dt$$
   where $\mathcal{C}$ is a curve with parameterization $\vec{r}(t)$
   (d) You should know how to show if a line integral is independent of path. And, if a line integral is independent of path, then evaluating the line integral depends only on the end points of the curve.
   (e) Applications of line integrals:
      i. From previous calc, you are aware that:
      $$\int_{\mathcal{C}} 1\,ds = \int_a^b \mid \vec{r}\,'(t) \mid dt$$
      gives the length of the curve $\mathcal{C}$ with parameterization $\vec{r}(t)$ for $a \le t \le b$.
      ii. Now, if $z = f(x, y) \ge 0$, on the curve $\mathcal{C}$, then
      $$\int_{\mathcal{C}} f\,ds$$
      gives the area of the *fence* above the curve $\mathcal{C}$ whose height is f.
   (f) Conservative vector fields. You should know how to check if a vector field, $\vec{F}$ in $\mathcal{R}^2$ or $\mathcal{R}^3$ is conservative or not.
   (g) If a vector field, $\vec{F}$, is conservative, then you should be able to:
      i. find a potential function, f, such that $\nabla f = \vec{F}$
      ii. apply the fundamental theorem of line integral to evaluate the line integral
   (h) If the curve $\mathcal{C}$ is a piece-wise smooth, simple, closed curve with a positive orientation, you should be able to apply *Green's* theorem to evaluate the line integral over the curve $\mathcal{C}$. If the region (D) enclosed by $\mathcal{C}$ is NOT a simply connected region, then you CANNOT

   apply *Green's* theorem directly, i.e. if (D) has holes, you cannot apply *Green's* theorem. You may have to decompose the curve $\mathcal{C}$ into several curves, each of which has the property that the region it encloses is simply connected.
   (i) You should know how to find the *curl* of a vector field in $\mathcal{R}^3$ and the *divergence* of a vector field in $\mathcal{R}^3$, and know how to apply their properties to solve problems.
   (j) You should know how to apply the results and the theorem to solve problems.

4. **Please** review all your assignments, the problems I did, and your notes.