

MATH 1101/2 (Ng/Fall 2004)
Handout for Parametric & Polar Equations
December 6, 2004.
Problems for Parametric & Polar Equations

- a. For the following problems, (i) use Mathematica to obtain the curves defined by the given parametric equations, and (ii) eliminate the parameter to find the Cartesian (x & y) equation of the curves.

1. $\{x = 4t^2; y = 16t\}$
2. $\{x = 2 \sin t; y = 3 \cos t\}$
3. $\{x = t + 1; y = t^3 + 1\}$
4. $\{x = \sqrt{t}; y = 1 - t\}$
5. $\{x = \sin^2 t; y = \cos^2 t\}$
6. $\{x = 3 \sec t; y = 2 \tan t\}$
7. $\{x = e^t; y = e^{-t}\}$
8. $\{x = e^t; y = e^t\}$
9. $\{x = e^t; y = \sqrt{t} \text{ for } 0 \leq t \leq 1\}$
10. $\{x = \frac{1-t}{1+t}; y = t^2 \text{ for } 0 \leq t \leq 1\}$

- b. For the following problems, use Mathematica to obtain the curves defined by the given parametric equations.

1. $\{x = \cos^2 t; y = \sin t\}$
2. $\{x = t(t^2 - 3); y = 3(t^2 - 3)\}$
3. $\{x = \frac{3t}{1+t^3}; y = \frac{t^2}{1+t^3}\}$
4. $\{x = \cos t; y = \sin t \cos t\}$

- c. For each of the following points in polar coordinates, i.e. (r, θ) , find its corresponding location point in rectangular Cartesian (x & y) coordinates.

1. $(r, \theta) = (1, \frac{\pi}{2})$
2. $(4, -\frac{2\pi}{3})$
3. $(-2, \frac{3\pi}{2})$
4. $(\sqrt{2}, \frac{\pi}{4})$
5. $(-4, -\frac{7\pi}{6})$

- d. For each of the following points in rectangular Cartesian coordinates, i.e. (x, y) , find a corresponding location point in polar coordinates, (r, θ) .

1. $(x, y) = (-1, 1)$
2. $(-1, -\sqrt{3})$
3. $(2\sqrt{3}, -2)$
4. $(3, 4)$

e. Use Mathematica to obtain the following curves defined in polar equations, i.e. use either *PolarPlot*.

Do not forget to load the graphics packages before executing the commands.

1. $r = 5$
2. $\theta = -\frac{\pi}{4}$
3. $r = 2(1 + \sin \theta)$
4. $r = 3(1 + \cos \theta)$
5. $r = 2(1 - \sin \theta)$
6. $r = 3(1 - \cos \theta)$
7. $r^2 = 2 \cos(2\theta)$ (lemniscate)
8. $r^2 = \sin(2\theta)$
9. $r = 2 \sin(\theta)$
10. $r = \cos(\theta) - \sin(\theta)$
11. $r = \cos(\theta) - \sin(\theta)$
12. $r = \theta$ for $\theta \geq 0$ (spiral)
13. $r = \frac{1}{\theta}$ for $\theta > 0$ (reciprocal spiral)
14. $r = e^\theta$ (logarithmic spiral)
15. $r = \sin(2\theta)$
16. $r = \sin(3\theta)$
17. $r = 2 \cos(3\theta)$
18. $r = 2 \cos(4\theta)$
19. Sketch $r = \cos(n\theta)$ for $n = 1, 2, 3, 4, 5, 6$
20. Sketch $r = \sin(n\theta)$ for $n = 1, 2, 3, 4, 5, 6$
21. $r = 2 + e^{-3\theta}$
22. $r = 2 - e^{-3\theta}$