Precalculus: The Twelve Basic Functions

- **Identity Function:** $y = f(x) = x$
- **Squaring Function:** $y = f(x) = x^2$
- **Cubing Function:** $y = f(x) = x^3$
- **Inverse Function:** $y = f(x) = \frac{1}{x}$
- **Square Root Function:** $y = f(x) = \sqrt{x}$
- **Exponential Function:** $y = f(x) = e^x$
- **Natural Logarithmic Function:** $y = f(x) = \ln(x)$
- **Sine Function:** $y = f(x) = \sin(x)$
- **Cosine Function:** $y = f(x) = \cos(x)$
- **Absolute Value Function:** $y = f(x) = |x|$
- **Greatest Integer Function:** $y = f(x) = \int(x)$
- **Logistic Function:** $y = f(x) = \frac{1}{1 + e^{-x}}$
**Concepts:** Familiarity with graphs of the 12 Basic Functions, and determining the properties of these functions from their graphs.

**Questions:** For the 12 Basic functions, use the sketches to answer the following (we will focus on the algebraic properties in the coming weeks).

1. Which have domain $x \in \mathbb{R}$ (another way of saying this is domain is $x \in (-\infty, \infty)$, or $-\infty < x < \infty$)?
   
   \[ f(x) = x, \; f(x) = x^2, \; f(x) = x^3, \; f(x) = e^x, \; f(x) = \sin(x), \; f(x) = \cos(x), \; f(x) = |x|, \; f(x) = \text{int}(x), \; f(x) = \frac{1}{1 + e^{-x}} \]

2. Which have domain $x \in [0, \infty)$ (domain $0 \leq x < \infty$)?
   
   \[ f(x) = \sqrt{x} \]

3. Which have domain $x \in (0, \infty)$ (domain $0 < x < \infty$)?
   
   \[ f(x) = \ln(x) \]

4. Which have range $y \in \mathbb{R}$ (range $-\infty < y < \infty$)?
   
   \[ f(x) = x, \; f(x) = x^3, \; f(x) = \ln(x) \]

5. Which have range $y \in [0, \infty)$ (range $0 \leq y < \infty$)?
   
   \[ f(x) = x^2, \; f(x) = \sqrt{x}, \; f(x) = |x| \]

6. Which have range $y \in (0, \infty)$ (range $0 < y < \infty$)?
   
   None.

7. Which have range $y \in (-\infty, 0) \cup (0, \infty)$ (range $-\infty < y < 0$ union with $0 < y < \infty$, which means $y \neq 0$)?
   
   \[ f(x) = \frac{1}{x} \]

8. Which have range $y \in (0, 1)$ (range $0 < y < 1$)?
   
   \[ f(x) = \frac{1}{1 + e^{-x}} \]

9. Which have vertical asymptotes?
   
   \[ f(x) = \frac{1}{x} \text{ has a vertical asymptote at } x = 0 \]

Aside: More formally, we may write:

\[ \lim_{x \to 0^+} \left( \frac{1}{x} \right) = \infty \text{ (read “the limit as } x \text{ approaches 0 from the right of } 1/x \text{ is infinity”) } \]

\[ \lim_{x \to 0^-} \left( \frac{1}{x} \right) = -\infty \text{ (“the limit as } x \text{ approaches 0 from the left of } 1/x \text{ is minus infinity”}. \]

We will talk about these right and left handed limits more in the coming weeks. I am including them here to help us get comfortable with the notation.

\[ f(x) = \ln(x) \text{ has a vertical asymptote at } x = 0 \]

\[ \lim_{x \to 0^+} \ln(x) = -\infty \text{ (read “the limit as } x \text{ approaches 0 from the right of } \ln(x) \text{ is minus infinity”) } \]
10. Which have horizontal asymptotes?

\[ f(x) = \frac{1}{x} \] has a horizontal asymptote at \( y = 0 \)

\[ \lim_{x \to \infty} \left( \frac{1}{x} \right) = 0 \] (read “the limit as \( x \) approaches infinity of \( 1/x \) is zero”)

\[ \lim_{x \to -\infty} \left( \frac{1}{x} \right) = 0 \] (read “the limit as \( x \) approaches minus infinity of \( 1/x \) is zero”)

\[ f(x) = e^x \] has a horizontal asymptote at \( y = 0 \)

\[ \lim_{x \to -\infty} (e^x) = 0 \] (read “the limit as \( x \) approaches minus infinity of \( e^x \) is zero”)

\[ f(x) = \frac{1}{1 + e^{-x}} \] has a horizontal asymptote at \( y = 0 \) and \( y = 1 \)

\[ \lim_{x \to \infty} \left( \frac{1}{1 + e^{-x}} \right) = 1 \] (read “the limit as \( x \) approaches infinity of \( 1/(1+e^{-x}) \) is one”)

\[ \lim_{x \to -\infty} \left( \frac{1}{1 + e^{-x}} \right) = 0 \] (read “the limit as \( x \) approaches minus infinity of \( 1/(1+e^{-x}) \) is zero”)

11. Which have local extrema?

\[ f(x) = x^2 \] has a global minimum of \( y = 0 \) at \( x = 0 \)

\[ f(x) = \sqrt{x} \] has a global minimum of \( y = 0 \) at \( x = 0 \)

\[ f(x) = \sin(x) \] has an infinite number of global minimums and maximums

\[ f(x) = \cos(x) \] has an infinite number of global minimums and maximums

\[ f(x) = |x| \] has a global minimum of \( y = 0 \) at \( x = 0 \)

12. Which are bounded below?

\[ f(x) = x^2, \; f(x) = \sqrt{x}, \; f(x) = e^x, \; f(x) = \sin(x), \; f(x) = \cos(x), \; f(x) = |x|, \; f(x) = \frac{1}{1 + e^{-x}} \]

13. Which have discontinuities?

\[ f(x) = \frac{1}{x}, \; f(x) = \text{int}(x) \]

14. Which are even (\( f(-x) = f(x) \) for all \( x \) in domain)?

\[ f(x) = x^2, \; f(x) = \cos(x), \; f(x) = |x| \]

15. Which are odd (\( f(-x) = -f(x) \) for all \( x \) in domain)?

\[ f(x) = x, \; f(x) = x^3, \; f(x) = \frac{1}{x}, \; f(x) = \sin(x) \]

16. Which are increasing over the interval \( x \in (-\infty, 0) \) (increasing for \(-\infty < x < 0\))? 

\[ f(x) = x, \; f(x) = x^3, \; f(x) = e^x, \; f(x) = \frac{1}{1 + e^{-x}} \]