You must remember to check your solutions and eliminate any extraneous solutions!

Questions

1. Solve $12 + \sqrt{4x + 5} = 7$.
2. Solve $y - \sqrt{y - 3} = 5$.
3. Solve $\sqrt{2y - 4} + 2 = y$.
4. Solve $\sqrt{3 - 5x} = 2$.
5. Solve $\sqrt{8x + 17} = \sqrt{2x + 8} + 3$.
6. Solve $\sqrt{2x + 9} - \sqrt{x + 1} = 2$.
7. In geology, the water depth $d$ near a mid-ocean spreading ridge depends on the square root of the distance $x$ from the ridge axis according to the relation $d = d_0 + a\sqrt{x}$, where $d_0$ is the depth of the ridge axis and $a$ is some constant. Solve the equation $d = d_0 + a\sqrt{x}$ for $x$.
8. Solve Graham’s law of effusion (used in molecular chemistry) $\frac{\rho_1}{\rho_2} = \sqrt{\frac{M_2}{M_1}}$ for $M_2$, then solve for $M_1$. 

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Solutions

Technique: Isolate a radical expression; square both sides of equation; solve for unknown; eliminate extraneous solutions.

1. 

\[12 + \sqrt{4x + 5} = 7\]

\[(\sqrt{4x + 5})^2 = (-5)^2\]

\[4x + 5 = 25\]

\[4x = 20\]

\[x = 5\]

Check for Extraneous Solutions:

\[x = 5 : \quad 12 + \sqrt{4(5) + 5} = 12 + \sqrt{25} = 12 + 5 = 17 \neq 7\]

So \(x = 5\) is extraneous, and the original equation has no solution.

2. 

\[y - \sqrt{y - 3} = 5\]

\[(y - 5)^2 = (\sqrt{y - 3})^2\]

\[y^2 - 10y + 25 = y - 3\]

\[y^2 - 11y + 28 = 0 \text{ Factor: Two numbers whose sum is } -11 \text{ product is } 28: -7, -4\]

\[(y - 7)(y - 4) = 0\]

\[y - 7 = 0 \text{ or } y - 4 = 0\]

\[y = 7 \text{ or } y = 4\]

Check for Extraneous Solutions:

\[y = 7 : \quad (7) - \sqrt{7 - 3} = 7 - \sqrt{4} = 7 - 2 = 5\]

\[y = 4 : \quad (4) - \sqrt{4 - 3} = 4 - \sqrt{1} = 3 \neq 5\]

So \(y = 7\) is the only solution to the original equation.

3. 

\[\sqrt{2y - 4} + 2 = y\]

\[(\sqrt{2y - 4})^2 = (y - 2)^2\]

\[2(y - 2) = (y - 2)^2\] Need to factor.

\[2(y - 2) - (y - 2)^2 = 0\]

\[(y - 2)(2 - (y - 2)) = 0\]

\[(y - 2)(2 + y - 2) = 0\]

\[(y - 2)(4 - y) = 0\]

\[y - 2 = 0 \text{ or } 4 - y = 0\]

\[y = 2 \text{ or } y = 4\]

Check for Extraneous Solutions:

\[y = 2 : \quad \sqrt{2(2) - 4} + 2 = 2\]

\[y = 4 : \quad \sqrt{2(4) - 4} + 2 = \sqrt{4} + 2 = 4\]

So both \(y = 2\) and \(y = 4\) are solutions.
4. Since we have a cube root, we cube both sides of the equation here.

\[(\sqrt[3]{3 - 5x})^3 = (2)^3\]

\[3 - 5x = 8\]

\[-5x = 5 \Rightarrow x = -1\]

Check for Extraneous Solutions:

\[x = -1 : \quad \sqrt[3]{3 - 5(-1)} = \sqrt[3]{8} = 2\]

So \(x = -1\) is a solution.

5.

\[(\sqrt{8x + 17})^2 = (\sqrt{2x + 8} + 3)^2\]

\[8x + 17 = 2x + 8 + 9 + 6\sqrt{2x + 8}\]

\[6x = 6\sqrt{2x + 8}\]

\[x = \sqrt{2x + 8}\]

\[(x)^2 = (\sqrt{2x + 8})^2\]

\[x^2 = 2x + 8\]

\[x^2 - 2x - 8 = 0\]

\[(x - 4)(x + 2) = 0\]

\[x - 4 = 0\] or \(x + 2 = 0\)

\[x = 4\] or \(x = -2\)

Check for Extraneous Solutions:

\[x = 4 : \quad \sqrt{8(4) + 17} = \sqrt{2(4) + 8 + 3} \Rightarrow \sqrt{49} = \sqrt{16} + 3 \Rightarrow 7 = 7 \text{ True}\]

\[x = -2 : \quad \sqrt{8(-2) + 17} = \sqrt{2(-2) + 8 + 3} \Rightarrow \sqrt{1} = \sqrt{4} + 3 \Rightarrow 1 = 5 \text{ False}\]

So \(x = 4\) is the only solution.

6.

\[\sqrt{2x + 9} - \sqrt{x + 1} = 2\]

\[(\sqrt{2x + 9})^2 = (2 + \sqrt{x + 1})^2\]

\[2x + 9 = 4 + 4\sqrt{x + 1} + (x + 1)\]

\[2x + 9 = 5 + x + 4\sqrt{x + 1}\]

\[x + 4 = 4\sqrt{x + 1}\]

\[(x + 4)^2 = (4\sqrt{x + 1})^2\]

\[x^2 + 8x + 16 = 16(x + 1)\]

\[x^2 + 8x + 16 = 16x + 16\]

\[x^2 - 8x = 0\]

\[x(x - 8) = 0\]

\[x = 0 \text{ or } x = 8\]

Check for Extraneous Solutions:

\[x = 0 : \quad \sqrt{2(0) + 9} - \sqrt{(0) + 1} = 3 - 1 = 2 \text{ True}\]

\[x = 8 : \quad \sqrt{2(8) + 9} - \sqrt{(8) + 1} = 5 - 3 = 2 \text{ True}\]

So both \(x = 0\) and \(x = 8\) are solutions.
7. 
\[ d = d_0 + a \sqrt{x} \]
\[ d - d_0 = a \sqrt{x} \]
\[ \frac{d - d_0}{a} = \sqrt{x} \]
\[ \left( \frac{d - d_0}{a} \right)^2 = (\sqrt{x})^2 \]
\[ \left( d - d_0 \right)^2 = x \]

8. 
\[ \left( \frac{\rho_1}{\rho_2} \right)^2 = \left( \sqrt{\frac{M_2}{M_1}} \right)^2 \]
\[ \left( \frac{\rho_1}{\rho_2} \right)^2 = \frac{M_2}{M_1} \]
\[ \left( \frac{\rho_1}{\rho_2} \right)^2 M_1 = M_2 \Rightarrow M_2 = \frac{M_1 \rho_1^2}{\rho_2^2} \]
\[ \left( \frac{\rho_1}{\rho_2} \right)^2 = \left( \sqrt{\frac{M_2}{M_1}} \right)^2 \]
\[ \rho_1^2 \quad M_2 = \frac{M_2}{M_1} \quad \rho_2^2 \]
\[ \rho_1^2 \quad M_1 = \frac{M_1}{M_2} \quad \rho_2^2 \]
\[ \frac{\rho_2^2}{\rho_1^2} M_2 = M_1 \Rightarrow M_1 = \frac{M_2 \rho_2^2}{\rho_1^2} \]