Questions

1. Solve \( x^2 - x - 20 = 0 \).
2. Solve \( x^2 + 11x + 18 = 0 \).
3. Solve \( 8x^2 = 72 \).
4. Solve \( (x - 5)(x + 4) = 2(x - 5) \).
5. Solve \( \frac{x^2 + 5x}{6} = 4 \).
6. Solve \( \frac{12x^2 - 4x}{5} = 8 \).
7. The area of a rectangular garden is 140 square meters. The width is 3 meters longer than one-half of the length. Find the length and width of the garden.
8. Jules is standing on a platform 6 meters high and throws a ball straight up as high as he can at a velocity of 13 meters per second. At what time \( t \) will the ball hit the ground? How far from the ground is the ball 2 seconds after Jules threw the ball (assume the ball is 6 meters from the ground when it leaves Jules’ hand).

Solutions

1. 
   \[ x^2 - x - 20 = 0 \]
   Find two numbers product is \(-20\) and sum is \(-1\): \(-5, 4\).
   \[
   (x - 5)(x + 4) = 0 \quad \text{Use Zero Factor Property.}
   
   (x - 5) = 0 \text{ or } (x + 4) = 0 \quad \text{Solve each linear equation.}
   
   x = 5 \text{ or } x = -4
   
   \text{Check:}
   
   (5)² - (5) - 20 = 25 - 25 = 0
   
   (-4)² - (-4) - 20 = 16 - 16 = 0

2. 
   \[ x^2 + 11x + 18 = 0 \]
   Find two numbers product is \(18\) and sum is \(11\): \(2, 9\).
   \[
   (x + 2)(x + 9) = 0 \quad \text{Factor.}
   
   (x + 2) = 0 \text{ or } (x + 9) = 0
   
   x = -2 \text{ or } x = -9
   
   \text{Check:}
   
   (-2)² + 11(-2) + 18 = 4 - 22 + 18 = 0
   
   (-9)² + 11(-9) + 18 = 81 - 99 + 18 = 0

3. 
   \[ 8x^2 - 72 = 0 \quad \text{Factor.} \]
   \[ 8(x^2 - 9) = 0 \]
   \[ x^2 - 9 = 0 \quad \text{Divide by 8. Difference of Squares.} \]
   \[
   (x + 3)(x - 3) = 0
   
   (x + 3) = 0 \text{ or } (x - 3) = 0
   
   x = -3 \text{ or } x = 3 \]
Check:

\[ 8(-3)^2 = 8(9) = 72 \]
\[ 8(3)^2 = 8(9) = 72 \]

Alternate solution, which only works because there was no \( x \) term:

\[ 8x^2 = 72 \]
\[ x^2 = 9 \]
\[ \sqrt{x^2} = \pm \sqrt{9} \]
\[ x = \pm 3 \]

4. Start by multiplying everything to get in form \( ax^2 + bx + c = 0 \).

\[
\begin{align*}
(x - 5)(x + 4) &= 2(x - 5) \\
x^2 - x - 20 &= 2x - 10 \\
x^2 - 3x &= 0 \\
x^2 - 20 - 2x &= 0 \\
x^2 - 3x - 10 &= 0
\end{align*}
\]

Find two numbers product is \(-10\) and sum is \(-3\): \(-5, 2\).

\[
\begin{align*}
(x - 5)(x + 2) &= 0 \\
(x - 5) &= 0 \text{ or } (x + 2) = 0 \\
x &= 5 \text{ or } x = -2
\end{align*}
\]

Check:

\[
\begin{align*}
((5) - 5)((5) + 4) - 2((5) - 5) &= 0 \\
((-2) - 5)((-2) + 4) - 2((-2) - 5) &= -14 + 14 = 0
\end{align*}
\]

5. Start by multiplying everything to get in form \( ax^2 + bx + c = 0 \).

\[
\begin{align*}
\frac{x^2 + 5x}{6} &= 4 \\
x^2 + 5x &= 24 \\
x^2 + 5x - 24 &= 0
\end{align*}
\]

Find two numbers product is \(-24\) and sum is \(5\): \(8, -3\).

\[
\begin{align*}
(x + 8)(x - 3) &= 0 \\
(x + 8) &= 0 \text{ or } (x - 3) = 0 \\
x &= -8 \text{ or } x = 3
\end{align*}
\]

Check:

\[
\begin{align*}
\frac{(-8)^2 + 5(-8)}{6} &= \frac{64 - 40}{6} = \frac{24}{6} = 4 \\
\frac{(3)^2 + 5(3)}{6} &= \frac{9 + 15}{6} = \frac{24}{6} = 4
\end{align*}
\]
6. Start by multiplying everything to get in form $ax^2 + bx + c = 0$.

$$\frac{12x^2 - 4x}{5} = 8$$

$$12x^2 - 4x = 40$$

$$3x^2 - x - 10 = 0$$

Grouping Method: Find two numbers product is $-30$ and sum is $-1$: $-6, 5$.

$$3x^2 - 6x + 5x - 10 = 0$$

Factor by grouping.

$$3(x - 2) + 5(x - 2) = 0$$

$$(3x + 5)(x - 2) = 0$$

$$(3x + 5) = 0 \text{ or } (x - 2) = 0$$

$$x = \frac{-5}{3} \text{ or } x = 2$$

Check:

$$\frac{12(-5/3)^2 - 4(-5/3)}{5} = \frac{12(25/9) + 20/3}{5} = \frac{100/3 + 20/3}{5} = \frac{120/3}{5} = \frac{40}{5} = 8$$

$$\frac{12(2)^2 - 4(2)}{5} = \frac{48 - 8}{5} = \frac{40}{5} = 8$$

7. Let $x$ be the length (in meters). Then the width is $\frac{x}{2} + 3$ meters. Area is 140 m$^2$.

Area = (length)(width)

$$140 = x \left(\frac{x}{2} + 3\right)$$

$$140 = \frac{x^2}{2} + 3x$$

write in form $ax^2 + bx + c = 0$.

$$280 = x^2 + 6x$$

$$0 = x^2 + 6x - 280$$

Find two numbers product is 6 and sum is $-280$: $-14, 20$.

$$(x - 14)(x + 20) = 0$$

$$(x - 14) = 0 \text{ or } (x + 20) = 0$$

$$x = 14 \text{ or } x = -20$$

Exclude the $x = -20$ as unphysical (can’t have negative length). So The length is $x = 14$ meters. Width is 10 meters.

8. Set $h = 6$ and $v = 13$ in our model equation $S = -5t^2 + vt + h$ (see handout).

$$-5t^2 + 13t + 6 = 0$$

Ball hits ground when $S = 0$. Use Grouping Method to factor.

$$-5t^2 + 13t + 6 = 0$$

Find two numbers product is $-30$ and sum is 13: $15, -2$.

$$-5t^2 + 15t - 2t + 6 = 0$$

$$(-5t - 2)(t - 3) = 0$$

$$(-5t - 2) = 0 \text{ or } (t - 3) = 0$$

$$t = -2/5 \text{ or } t = 3$$

Exclude the $t = -5/3$ as unphysical, so the ball hits the ground after 3 seconds.

Two second after throwing the ball, it it $S = -5(2)^2 + 13(2) + 6 = -20 + 26 + 6 = 12$ meters above the ground.