Questions

1. Find the slope of the straight line that passes through the points (4, 1) and (6, 7).
2. Find the slope of the straight line that passes through the points (11, 2) and (5, 14).
3. Find the slope of the straight line that passes through the points (−6, −5) and (2, −7).
4. Write the equation for a straight line in slope-intercept form with slope \( m = \frac{2}{3} \) and \( y \)-intercept \( (0, 5) \).
5. Write the equation for a straight line in slope-intercept form with slope \( m = 5 \) and \( y \)-intercept \( (0, −6) \).
6. Write the equation for a straight line in slope-intercept form with slope \( m = \frac{2}{3} \) and \( y \)-intercept \( (0, 1/2) \).
7. Sketch the straight line \( y = mx + b \) where \( m = \frac{1}{3} \) and \( b = −2 \).
8. Sketch the straight line \( y = mx + b \) where \( m = −\frac{3}{2} \) and \( b = 4 \).
9. Sketch the straight line \( y = 3x \).
10. A line has a slope of \( \frac{11}{4} \). What is the slope of a line parallel to it? What is the slope of a line perpendicular to it?
11. A line has equation \( y = \frac{3}{5}x − 5 \). What is the slope of a line parallel to it? What is the slope of a line perpendicular to it?
12. During the years from 1980 to 2005 the total income for the U.S. federal budget can be approximated by the equation \( y = 14(4x + 35) \), where \( x \) is the number of years since 1980 and \( y \) is the amount of money in billions of dollars (source: U.S. Office of Management and Budget).

Write the equation in slope-intercept form. Find the slope and \( y \)-intercept. What is the meaning of the slope in this situation?
Solutions

1. slope = \frac{\Delta y}{\Delta x} = \frac{1 - 7}{4 - 6} = \frac{-6}{-2} = 3.

4. \[ y = \frac{2}{3}x + 5. \]

2. slope = \frac{\Delta y}{\Delta x} = \frac{2 - 14}{11 - 5} = \frac{-12}{6} = -2.

5. \[ y = 5x - 6. \]

3. slope = \frac{\Delta y}{\Delta x} = \frac{-5 - (-7)}{-6 - 2} = \frac{2}{-8} = \frac{-1}{4}.

6. \[ y = \frac{2}{3}x + \frac{1}{2}. \]

7. \[ y = \frac{1}{3}x - 2. \]

8. \[ y = -\frac{3}{2}x + 4. \]

9. \[ y = 3x. \]

Intercept is \( b = 0 \). Slope is \( m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} \)

10. Parallel: \[ \frac{11}{4}. \] Perpendicular: \[ -\frac{4}{11}. \]

11. Parallel: \[ \frac{3}{5}. \] Perpendicular: \[ -\frac{5}{3}. \]

12. \[ y = 14(4x + 35) = 56x + 490 \Rightarrow \text{ slope } = 56 \text{ and } y\text{-intercept is } (0, 490). \]

The slope is the amount of increase in income of the federal budget in billions of dollars per year.

Aside: This equation is not as good as it could be, since \( x \) represents the number of years since 1980. The equation would be improved if the independent variable represented the year. We can make this change by introducing a change in variables.

Let \( z \) be the year. Then \( z = 1980 + x. \) Therefore, \( x = z - 1980. \) The equation becomes

\[ y = 56x + 490 \]
\[ y = 56(z - 1980) + 490 = 56z - 110,390 \]

What was the federal budget in 1987? Answer: \[ y = 56z - 110,390 = 56(1987) - 110,390 = 882 \text{ billion dollars}. \] This is the same answer you get if you use \( y = 56x + 490 \) with \( x = 7. \)