Questions

1. Solve for $x$ when $\frac{2}{3}x = \frac{1}{15}x + \frac{3}{5}$.

2. Solve for $x$ when $\frac{x}{2} + \frac{x}{5} = \frac{7}{10}$.

3. Solve for $x$ when $20 - \frac{1}{3}x = \frac{1}{2}x$.

4. Is 4 a solution to $\frac{1}{2}(y - 2) + 2 = \frac{3}{8}(3y - 4)$?

5. Solve for $x$ when $0.3x - 0.2(3 - 5x) = -0.5(x - 6)$.

6. Solve for $x$ when $\frac{4}{5}x - \frac{2}{3} = \frac{3x + 1}{2}$.

7. Solve for $x$ when $\frac{4}{7}x + \frac{1}{3} = \frac{3x - 2}{14}$.

8. Solve for $x$ when $-1 + 5(x - 2) = 12x + 3 - 7x$.

9. Solve for $x$ when $9(x + 3) - 6 = 24 - 2x - 3 + 11x$.

Solutions

1. The LCD (lowest common denominator) is 15, so multiply the equation by 15 to remove the fractions.

   \[
   \frac{2}{3}x = \frac{1}{15}x + \frac{3}{5} \\
   15 \cdot \left( \frac{2}{3}x \right) = 15 \cdot \left( \frac{1}{15}x + \frac{3}{5} \right) \\
   10x = 15 \cdot \frac{1}{15}x + 15 \cdot \frac{3}{5} \text{ distribute!} \\
   10x = x + 9 \text{ simplify} \\
   10x - x = x + 9 - x \text{ addition principle} \\
   9x = 9 \text{ simplify} \\
   \frac{1}{9} \cdot 9x = \frac{1}{9} \cdot 9 \text{ multiplication principle} \\
   x = 1 \text{ simplify}
   \]

2. LCD is 10.

   \[
   \frac{x}{2} + \frac{x}{5} = \frac{7}{10} \\
   10 \cdot \left( \frac{x}{2} + \frac{x}{5} \right) = 10 \cdot \frac{7}{10} \\
   10 \cdot \frac{x}{2} + 10 \cdot \frac{x}{5} = 7 \\
   5x + 2x = 7 \\
   7x = 7 \\
   \frac{1}{7} \cdot 7x = \frac{1}{7} \cdot 7 \\
   x = 1
   \]

3. LCD is 6.

   \[
   20 - \frac{1}{3}x = \frac{1}{2}x \\
   6 \cdot \left( 20 - \frac{1}{3}x \right) = 6 \cdot \frac{1}{2}x \\
   6 \cdot 20 - 6 \cdot \frac{1}{3}x = 3x \\
   120 - 2x = 3x \\
   120 - 2x + 2x = 3x + 2x \\
   120 = 5x \\
   \frac{1}{5} \cdot 120 = \frac{1}{5} \cdot 5x \\
   24 = x
   \]
4. You could substitute \( y = 4 \) to check, but I am going to solve it instead. LCD is 8.
\[
\frac{1}{2}(y - 2) + 2 = \frac{3}{8}(3y - 4)
\]
\[
8 \cdot \left( \frac{1}{2}(y - 2) + 2 \right) = 8 \cdot \frac{3}{8}(3y - 4)
\]
\[
8 \cdot \frac{1}{2}(y - 2) + 8 \cdot 2 = 3(3y - 4)
\]
\[
4(y - 2) + 16 = 9y - 12
\]
\[
4y - 8 + 16 = 9y - 12
\]
\[
4y + 8 = 9y - 12
\]
\[
4y + 8 - 9y - 8 = 9y - 12 - 9y - 8
\]
\[
-5y = -20
\]
\[
\frac{1}{5} (-5y) = \frac{1}{5} \cdot (-20)
\]
\[
y = 4
\]

5.
\[
0.3x - 0.2(3 - 5x) = -0.5(x - 6)
\]
\[
0.3x - 0.6 + x = -0.5x + 3
\]
\[
1.3x - 0.6 = -0.5x + 3
\]
\[
1.3x - 0.6 + 0.5x + 0.6 = -0.5x + 3 + 0.5x + 0.6
\]
\[
1.8x = 3.6
\]
\[
\frac{1}{1.8} \cdot 1.8x = \frac{1}{1.8} \cdot 3.6
\]
\[
x = 2
\]

6. LCD is 30.
\[
\frac{4}{5}x - \frac{2}{3} = \frac{3x + 1}{2}
\]
\[
30 \cdot \left( \frac{4}{5}x - \frac{2}{3} \right) = 30 \cdot \frac{3x + 1}{2}
\]
\[
30 \cdot \frac{4}{5}x - 30 \cdot \frac{2}{3} = 30 \cdot \frac{1}{2} (3x + 1)
\]
Note in above I wrote \( \frac{3x + 1}{2} \) as \( \frac{1}{2} \cdot (3x + 1) \). Doing this helps reduce errors!
\[
24x - 20 = 15 \cdot (3x + 1)
\]
\[
24x - 20 = 45x + 15
\]
\[
24x - 20 - 45x + 20 = 45x + 15 - 45x + 20
\]
\[
-21x = 35
\]
\[
\frac{1}{-21} \cdot (-21x) = \frac{1}{-21} \cdot 35
\]
\[
x = \frac{35}{21} = \frac{5}{3}
\]

7. LCD is 42.
\[
\frac{4}{7}x + \frac{1}{3} = \frac{3x - 2}{14}
\]
\[
4 \cdot \frac{4}{7}x + \frac{1}{3} = \frac{1}{4} (3x - 2)
\]
\[
42 \cdot \left( \frac{4}{7}x + \frac{1}{3} \right) = 42 \cdot \frac{1}{4} (3x - 2)
\]
\[
42 \cdot \frac{4}{7}x + 42 \cdot \frac{1}{3} = 3(3x - 2)
\]
\[
24x + 14 = 3(3x - 2)
\]
\[
24x + 14 = 9x - 6
\]
\[
24x + 14 - 9x - 14 = 9x - 6 - 9x - 14
\]
\[
15x = -20
\]
\[
\frac{1}{15} \cdot 15x = \frac{1}{15} \cdot (-20)
\]
\[
x = \frac{-20}{15} = -\frac{4}{3}
\]

8.
\[
-1 + 5(x - 2) = 12x + 3 - 7x
\]
\[
-1 + 5x - 10 = 5x + 3
\]
\[
5x - 9 - 5x = 5x + 3 - 5x
\]
\[
-9 = 3
\]
We have to interpret what we have found. Since -9 never equals 3, the equation is never true no matter what value of \( x \) we put in. This means the equation has no solution.

9.
\[
9(x + 3) - 6 = 24 - 2x - 3 + 11x
\]
\[
9x + 27 - 6 = 21 + 9x
\]
\[
9x + 21 = 21 + 9x
\]
\[
9x + 21 - 9x = 21 + 9x - 9x
\]
\[
21 = 21
\]
We have to interpret what we have found. Since 21 is always equal to 21, the equation is true for any value of \( x \) that we try. Therefore, there are an infinite number of solutions.