Can asymmetric conditional volatility imply asymmetric tail dependence?

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A B S T R A C T

In this article, we investigate two types of asymmetries, that is, the asymmetry of conditional volatility and the asymmetry of tail dependence in the crude oil markets. We employ the two different sample datasets in which each dataset covers the time period of stable and unstable oil prices, individually. A variety of different copulas and three asymmetric GARCH regression models are used in order to capture the two types of asymmetries. In particular, we extend the TBL-GARCH model proposed by Choi et al. (2012) to the asymmetric GARCH regression type model. The findings from the two different approaches are congruent, in that there is no asymmetry of tail dependence and no asymmetric conditional volatility in crude oil returns over the two different sample periods. Our study reconfirms the findings of Aboura and Wagner (2016) by showing that asymmetric conditional volatility relates to asymmetric tail dependence.

1. Introduction

Crude oil is one of the most important commodities affecting economic activities. In particular, a better understanding of crude oil markets would be crucial for portfolio allocation and risk management and asset pricing in practice. Crude oil markets closely move together. Thus, it is important to consider that financial market agents consider such co-movement, in particular, upper and lower tail dependencEs of oil prices across the different crude oil markets. The analysis of the dependence structure could allow them to have portfolio selection and hedging strategies. It is also crucial to understand the volatility of the oil prices because persistent changes in volatility can expose the crude oil market participants to risk.

The relationship between oil returns and trading volume has received considerable attention in the literature. Abba Abdullahi et al. (2014) investigate the impact of trading volume on crude oil returns for the WTI and Brent market, individually. They find the congruent results that neither returns nor trading volume contain any important information to forecast the variance of the other in either market from the variance decomposition analysis. Moosa et al. (2003) find bi-directional causality between the two variables in the WTI market. However, Bhar and Hamori (2005) show the unidirectional relationship by using the AR-GARCH model such that oil returns lead trading volume in the same oil market. Asche et al. (2003) also find the relationship between crude oil and refined product markets by using multivariate Johansen tests. Tong et al. (2012) find some evidence of the asymmetry in the propagation of crisis or bubble between crude oil and refined petroleum markets.

The purpose of this article is to propose an approach using the asymmetric conditional volatility that can imply asymmetric tail dependence. We reconfirms the findings of Aboura and Wagner (2016). By extending the model of Choi et al. (2012), we deliver a model setting, where asymmetric volatility relates to asymmetric tail dependence. In order to illustrate the economic implications of the effect, this study turns to an energy market application over two different sample periods for stable and unstable oil prices. We investigate two types of asymmetries: the asymmetry in tail dependence between West Texas Intermediate (WTI) and Brent crude oil returns and the asymmetry in volatility of the WTI returns conditional on the Brent returns.

Our contribution to the literature is twofold. First, our study provides reconfirmation of the findings in Aboura and Wagner (2016) by using relatively new techniques: copulas and asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) regression models. This article shows a clear relationship between the two apparently different asymmetries. Unlike Aboura and Wagner (2016), we consider asymmetric conditional volatility rather than asymmetric volatility by using three different asymmetric GARCH regression models. Second, we extend the Threshold-Bilinear GARCH (TBL-GARCH) proposed by Choi et al. (2012) to the asymmetric

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TBL-GARCH regression model. The method is simple, yet it is powerful and flexible enough to investigate the asymmetric conditional volatility.

The relationship between the asymmetry of conditional volatility and the asymmetry of tail dependence in the crude oil markets has important implications for hedgers and energy policymakers. Market participants can find available alternatives to hedge the possible risk, and regulators can anticipate their oil policy effectiveness in the market. We find symmetric tail dependence for crude oil returns. The link between crude oil returns indicates that the crude oil markets are highly intergraded. Also, dependence does not change in times of extreme upward or downward market movements, indicating that a shock in one market is fully transmitted to the other market. This finding provides important implications for risk management in that investments in the crude oil markets offer limited risk diversification because the markets boomed and crashed together. Energy policymakers should know that conditional volatility and tail dependence respond to the same information flow in the crude oil markets.

An extensive literature well documents both asymmetric volatility and asymmetric tail dependence in the equity and bond markets. For example, Junker et al. (2006) study dependence in the term structure of U.S. Treasury yields by using various copula functions. The transformed Frank copula shows best overall fit in their study. They find upper tail dependence and zero lower tail dependence in yield shocks. In addition, they show that normal copula function systematically provides substantial bias of up to 6% in the portfolio value-at-risk application. Conditional dependence is also examined in the literature such as U.S. dollar exchange rate returns (Patton, 2001) and international stock market returns (Rockinger and Jondeau, 2001).

Bekaert and Wu (2000) provide a framework to investigate volatility asymmetry at both the market and firm level simultaneously. Then, they apply the model to the portfolio level constructed from the Nikkei 225 index. The empirical application shows that the main determinant of asymmetric volatility in the equity market is volatility feedback instead of leverage effects. From their empirical findings, they argue that “the leverage effect” is a misnomer and “asymmetric volatility” would be more appropriate. There is also extensive literature on the study of the relationship between returns and conditional volatility (see Engle et al. (1987), Engle and Ng (1993) and Braun et al. (1995)).

Aboura and Wagner (2016) investigate asymmetric volatility for the S&P 500 and VIX index returns and document contemporaneous asymmetric effects. They provide empirical evidence that substantial equity market declines can be partially explained by extreme volatility feedback. However, they could not find directional causality between returns and volatility in either way. Their main finding is that the extreme asymmetric volatility effect exists, as they show volatility-return tail dependence during the market stress. Our study reconfirms this finding in that there is a relationship between the asymmetry of the conditional volatility and the asymmetry of tail dependence.

The existing studies investigate the link between oil returns and other financial asset returns, such as stock returns, exchange rate returns, and interest rates. For example, Chen and Chen (2007) show a cointegrating relationship between oil prices and exchange rates. Akram (2004) finds a non-linear relationship between oil prices and the Norwegian exchange rate. Wu et al. (2012) examine the tail dependence of crude oil and the U.S. dollar exchange rate. In addition, Cologni and Manera (2008) and Arora and Tanner (2013) provide empirical evidence of co-movement between oil prices and interest rates. The previous empirical literature presents the negative relationship between oil price returns and stock returns, such as Filis (2010), Chen (2010) and Miller and Ratti (2009). Sadorsky (1999) finds that oil price volatility has impacts on the U.S. real stock returns and Oberndorfer (2009) shows a similar effect of oil price volatility on European stock markets.

Batten et al. (2015) examine the degree of integration between financial markets and commodity markets by using an asset pricing framework. They find that emerging market investors benefit from positive risk adjusted returns for gold and rice markets. In addition, Gérard et al. (2003), Chi et al. (2006) and Jeon et al. (2006) study the degree of integration between stock and world markets. Basier and Sadowsky (2016) examine volatility dynamics, conditional correlations and hedge ratios between oil and other various assets, such as stock, VIX, gold, and bonds by using multivariate GARCH models. They find empirical evidence of a positive asymmetric volatility of oil prices. Also, their study suggests that oil is the most effective hedge for emerging market stock prices. Khalafoui et al. (2015) focus on the mean and volatility spillovers and hedging between WTI crude oil and stock market prices for the G-7 countries. Their empirical results show time-varying correlations and volatility spillover effects across markets. Arouri and Nguyen (2010) examine the short-term linkage between oil price changes and stock returns. They find that the reaction of stock returns to oil price shocks considerably varies across disaggregated sector level.

The more recent studies on the oil and stock market relationship include Balcilar et al. (2017) and Batten et al. (2017). Balcilar et al. (2017) investigate the short- and long-run co-movement of the S&P 500 and WTI oil prices. In particular, the framework of common cycles and common trends is employed. They find that the two financial assets share a common stochastic trend from 1859 to 2015, and common cycle exists only during the post-World War II period. Their empirical analysis also provides evidence that the short-run oil price is determined by transitory shocks, and the stock market is affected by permanent shocks in short- and long-runs. Batten et al. (2017) examine the link between a stock market and an energy portfolio, consisting of coal, natural gas, and oil. They find time-varying integration between individual stock markets and the energy portfolio in Asia. This finding implied that benefits can arise for Asian stock market investors by successfully hedging the common factor caused by energy price risk.

There is a limited amount of recent literature on using copulas to study dependence across financial markets. Chang (2012a) investigates both the interdependence between spot and futures returns and the individual dynamic process of the return series. Li and Yang (2013) attempt to find the relationships between the volatility of rubber futures and the oil index via the copula-based GARCH model. Other studies focus on the dependence structure and co-movements in stock markets using copulas. See Mensah and Alagidede (2017) for the dependence structure across African stock markets and Nguyen et al. (2016) for the dependence structure of dependency between gold and stock markets.

While the above literature focuses on the dependence structure and co-movements, we utilize the copula approach in order to assess asymmetry in the dependence structure between crude oil markets. The asymmetry of tail dependence in Uhm et al. (2012) is a case in which the level of dependence at the upper tail is not equal to the level of dependence at the lower tail. Tong et al. (2013) find asymmetry in tail dependence between crude and heating oil returns and between crude oil and jet fuel returns by using the asymmetric copulas. Reboredo (2011) employs various copula models with time-invariant and time-varying dependence structures and finds no evidence of asymmetric tail dependence between different crude oil spot market prices for WTI, Brent, Dubai, and Maya.

To study the asymmetry of tail dependence between WTI and Brent crude oil returns, we specify a joint model for dependence with various dependence structures. For example, the Normal copula has no tail dependence, the Clayton copula has lower tail dependence, the Plackett copula has symmetric tail dependence, the Frank copula has symmetric tail dependence, the Gumbel copula has upper tail dependence, the Student-t copula has symmetric tail dependence, and the symmetrized Joe-Clayton (SJJC) copula by Patton (2006) allows for asymmetric tail dependence and nest symmetry as a special case.

One of the well-established features in the financial time series is
the asymmetric volatility (Black, 1976; Engle and Ng, 1993). It is well-
known that the impact of a negative shock on volatility is stronger than
that of positive one. This characteristic is captured by the asymmetric
volatility. A number of recent empirical studies have been dedicated to
examining asymmetry in crude oil volatility. Fan et al. (2008) find a
spillover effect between WTI and Brent crude oil prices and an
asymmetric volatility in the WTI returns by using various specifications
of the GARCH models. Nomikos and Andriosopoulos (2012) also find
an asymmetric volatility for WTI spot markets and an inverse asym-
metric volatility for the natural gas spot prices. However, other empiri-

cal studies (e.g., Agnolucci, 2009; Cheong, 2009; Chang, 2012b) find no asymmetric volatility for the WTI market.

We examine the asymmetric conditional volatility of the WTI return
series given Brent oil prices by employing three different asymmetric
GARCH regression models, which are the Threshold GARCH (T-
GARCH), Bilinear GARCH (BL-GARCH), and TBL-GARCH regression
models. The T-GARCH model was proposed by Zakoian (1994) is typically
used to capture the leverage or asymmetric effect in the volatility by
introducing thresholds in the volatility equation. The BL-GARCH
model proposed by Storti and Vitale (2003) captures the asymmetry by
including the interactions between past observations and volatilities
in the volatility equation. Recently Choi et al. (2012) propose the
asymmetric GARCH model featuring both a threshold effect and
bilinear structure. In this article, we extend the TBL-GARCH to a
regression model in order to examine the asymmetric conditional
volatility. A comparative study between the three asymmetric GARCH
regression models is carried out. All alternative specifications consist-
tently show evidence that reveals no asymmetric conditional volatility,
while the best fitting model is BL-GARCH in our study.

The remainder of this paper is structured as follows. The next
section introduces the econometric methodologies, such as copula
methods and asymmetric GARCH regression models. In Section 3,
we discuss our empirical results regarding the asymmetry of tail
dependence and the asymmetric conditional volatility in oil prices.
Concluding remarks are presented in Section 4.

2. Econometric methodology

2.1. Copula methods

The dependence structure of a set of random variables is contained
within F. The idea of separating F into one part which describes the
dependence structure and other parts, which describe only the margin-
al behavior, has led to the concept of a copula. A copula is a
multivariate uniform distribution representing a way of trying to
extract the dependence structure of the random variables from the
joint distribution function. It is a useful approach to understand and
model dependent random variables. Let $F_X$ and $F_Y$ be marginal
distributions, then every joint distribution can be written as
$F_{XY}(x, y) = C(F_X(x), F_Y(y))$. A bivariate copula is a function
$C: [0, 1]^2 \to [0, 1]$, whose domain is the entire unit square with the
following three properties: We denote $u = F_X(x)$ and $v = F_Y(y)$.

- $C(u, 0) = C(0, v) = 0$, $\forall u, v \in [0, 1]$
- $C(u, 1) = C(1, v) = u$, $\forall u, v \in [0, 1]$
- $C(u_1, v_2) - C(u_1, v_1) - C(u_2, v_1) + C(u_2, v_2) \geq 0$, $\forall u_1, u_2, v_1, v_2 \in [0, 1]$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$. We denote $u_1 = F_X(x_1)$, $u_2 = F_X(x_2)$,
  $v_1 = F_Y(y_1)$, and $v_2 = F_Y(y_2)$.

The coefficients of upper and lower tail dependence for $t^U \in [0, 1]$ and $t^L \in [0, 1]$ of $(X, Y)$ are defined by Nelson (2006) as

$$t^L = \lim_{\epsilon \to 0} -\epsilon \log \mathbb{P}(U \leq \epsilon V \leq \epsilon) = \lim_{\epsilon \to 0} -\epsilon \log \mathbb{P}(V \leq \epsilon U \leq \epsilon)$$

and $t^U = \lim_{\epsilon \to 0} 1 - \epsilon \mathbb{P}(U > \epsilon V > \epsilon) = \lim_{\epsilon \to 0} 1 - \epsilon \mathbb{P}(V > \epsilon U > \epsilon)$. (1)

The Normal copula has $t^U = t^L = 0$, meaning that in the extreme tails
of the distribution, the variables are independent. The normal distri-
bution is the most common assumption in finance, but it does not have
tail-dependence while the Gumbel copula has right tail depend-
dence. The Gumbel survival copula has left tail dependence. The
Clayton copula has left tail dependence and contours that are quite a
bit more peaked for negative events than they are for joint positive
events.

Patton (2006) points out one major drawback of the Joe-Clayton copula: even when the two tail dependence measures are equal, there is still some (slight) asymmetry in the Joe-Clayton copula. For $\kappa = 1/\log(2 - t^L)$, $\gamma = -1/\log(2 - t^U)$, $t^L \in (0, 1)$, and $t^U \in (0, 1)$, the Joe-Clayton copula, Joe (1997) is

$$C_{JC}(u, v; t^L, t^U) = 1 - (1 - (1 - u)^{t^L}) + [1 - (1 - v)^{t^U}] - 1^{t^L/t^U}$$

To investigate asymmetry of dependence by using both tail dependencies,
Patton (2006) proposes the symmetrized Joe-Clayton (SJC) copula:

$$C_{SJC}(u, v; t^L, t^U) = 0.5 \times \left[ C_{JC}(u, v; t^L, t^U) + C_{JC}(1 - u, 1 - v; t^L, t^U) + u + v - 1 \right].$$

A copula model typically used in econometrics is the time-varying

normal copula:

$$C^G(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\sigma(1 - \rho^2)} \exp \left\{ \frac{-(u^2 - 2\rhoux + \sigma^2)}{2(1 - \rho^2)} \right\} dr d\sigma,$$

where $\Phi^H$ is the inverse of the standard normal distribution function
and

$$\beta_i = \tilde{\Lambda} \left\{ a_0 + a_1 \rho_{i-1} + a_2 \sum_{j=1}^{10} \Phi^{-1}(\alpha_{i-j}) \Phi^{-1}(\alpha_{i-j}) \right\},$$

where $\tilde{\Lambda} = (1 - e^{-\gamma})(1 + e^{-\gamma})^{-1} = \tanh(\gamma/2)$ is the modified logistic
transformation, as used in Patton (2006).  

2.2. Asymmetric GARCH regression models

The main source of asymmetry in the volatility of the financial time
series is related to the so-called leverage effect first noted by Black
(1976). As shown in many empirical studies, positive and negative
innovations have different impacts on future volatility. There is a long
list of variations of GARCH models that consider the asymmetry. In
this article, we study the existence of asymmetric conditional volatility
by using asymmetric GARCH regression models for WTI oil, given
Brent oil prices. For the asymmetric GARCH regression model, we have
three different asymmetric GARCH specifications: the T-GARCH model
(Glosten et al., 1993; Zakoian, 1994), which considers a threshold
effect in modeling volatility, the BL-GARCH model (Storti and Vitale,
2003) with the bilinear structure, and the TBL-GARCH model (Choi et al.,
2012), which combines the threshold effect with the bilinear
structure in order to generate a broader class of asymmetric GARCH
models. For more details about the asymmetric GARCH models, see
Engle and Ng (1993).

Let $\varepsilon_t$ be independent and identically distributed random variables
with zero mean and unit variance, for $t = 1, 2, ..., T$. For $\alpha_0 > 0, \alpha_1, \alpha_2 \geq 0, \epsilon^c = \max(0, \epsilon), \epsilon^- = \max(0, -\epsilon)$, and $\text{le} = \epsilon^c + \epsilon^-$, and $\beta_i \geq 0$, the AR(1)-T-GARCH(1, 1) regression model is expressed as

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Summary statistics for Brent and WTI.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Brent</td>
<td>0.040</td>
</tr>
<tr>
<td>WTI</td>
<td>-0.004</td>
</tr>
<tr>
<td>Minimum</td>
<td>-3.827</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.827</td>
</tr>
<tr>
<td>St.D</td>
<td>1.079</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.111</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.687</td>
</tr>
</tbody>
</table>

Note: The table records the various descriptive statistics for Brent and WTI oil returns, individually. St.D refers to the standard deviation. Sample period: January 3, 2013 to October 6, 2014.
Fig. 1. Daily log-returns of Brent. Note: The left figure in the upper panel displays the fluctuation of the Brent oil daily returns. The right figure in the upper panel and the left figure in the lower panel exhibit ACF and PACF plots, respectively. The horizontal axis of the plots indicates the lag at which the (partial) autocorrelation is computed, and the vertical axis indicates the correlation values. The right figure in the lower panel displays the McLeod-Li test results for the presence of heteroscedascity. The \( p \)-values indicate the Ljung-Box test statistics. Sample period: January 3, 2013 to October 6, 2014.

Fig. 2. Daily log returns of WTI. Note: The left figure in the upper panel displays the fluctuation of the WTI oil daily returns. The right figure in the upper panel and the left figure in the lower panel exhibit ACF and PACF plots, respectively. The horizontal axis of the plots indicates the lag at which the (partial) autocorrelation is computed, and the vertical axis indicates the correlation values. The right figure in the lower panel displays the McLeod-Li test results for the presence of heteroscedascity. The \( p \)-values indicate the Ljung-Box test statistics. Sample period: January 3, 2013 to October 6, 2014.

Table 2
Granger-Causality Wald test by VAR(1) model.

<table>
<thead>
<tr>
<th>Causality</th>
<th>Brent → WTI</th>
<th>WTI → Brent</th>
</tr>
</thead>
<tbody>
<tr>
<td>DF</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Chi-square</td>
<td>13.67</td>
<td>0.89</td>
</tr>
<tr>
<td>( p )-value</td>
<td>0.000</td>
<td>0.346</td>
</tr>
</tbody>
</table>

Note: The table reports the results of the Granger-Causality Wald test by the VAR(1) model between Brent and WTI oil prices. Brent oil returns do Granger cause WTI oil returns at even 1% significance level, but the WTI returns do not significantly Granger cause the change of WTI oil returns. DF refers to the degrees of freedom. Sample period: January 3, 2013 to October 6, 2014.

Table 3

<table>
<thead>
<tr>
<th>( H_0 ): Rank=m</th>
<th>( H_1 ): Rank=s</th>
<th>Eigenvalue</th>
<th>Filter</th>
<th>5% Critical Value</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.588</td>
<td>−178.38</td>
<td>−14.10</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.594</td>
<td>−175.89</td>
<td>−8.80</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.495</td>
<td>−218.54</td>
<td>−23.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table provides the test results for Stock-Watson’s common trends, which is useful for examining cointegration relationships. It shows that there are 2 cointegrated I (1) series with cointegration rank 1. Therefore, our return series have a \( 2 - 1 = 1 \) common trend. Sample period: January 3, 2013 to October 6, 2014.
Mean equation: \[ Y_t = a + b \times X_t + \epsilon_t, \quad \epsilon_t = \phi_1 \times \epsilon_{t-1}. \]

\[ \epsilon_{t-1} = \sqrt{h_{t-1}} \epsilon_{t-1}. \]

Variance equation: \[ h_t = a_0 + a_1 \times (\epsilon_{t-1}^2) + a_2 \times (\epsilon_{t-2}^2) + b_1 \times h_{t-1}. \]

The case of \( a_1 < a_2 \) corresponds to an asymmetric volatility frequently observed in T-GARCH(1, 1) (Choi et al., 2012). Similarly, for \( a_0 > 0, a_1 \geq 0, -\infty < \gamma < \infty, \) and \( \beta_1 \geq 0, \) the AR(1)-BL-GARCH(1, 1) regression model is expressed as

Mean equation: \[ Y_t = a + b \times X_t + \epsilon_t, \quad \epsilon_t = \phi_1 \times \epsilon_{t-1}. \]

\[ \epsilon_{t-1} = \sqrt{h_{t-1}} \epsilon_{t-1}. \]

Variance equation: \[ h_t = a_0 + a_1 \times (\epsilon_{t-1}^2) + a_2 \times (\epsilon_{t-2}^2) + b_1 \times h_{t-1} + \gamma \times \epsilon_{t-1} \times \sqrt{h_{t-1}}. \]

(2)

(3)

An asymmetric volatility corresponds to \( \beta < 0 \) and the BL-GARCH model is known to be capable of accommodating shift-features in \( h_t \) (Choi et al., 2012), and the asymmetric volatility is explained by the interactions between past observations and volatilities. Thus, the BL-GARCH model is a useful tool for modeling time varying conditional variances and asymmetric conditional volatility in financial time series (Storti and Vitale, 2003). In this study, we extend the TBL-GARCH to a TBL-GARCH regression type model in order to examine the asymmetric conditional volatility of WTI oil prices given Brent oil prices. For \( a_0 > 0, a_1, a_2 \geq 0, \) \( c^- = \max(0, c^+), \) \( c^- = \max(0, c^+), \) and \( \epsilon = c^+ - c^- \) and \( |\epsilon| = c^+ + c^- - \infty < \beta_1 < \infty, \) and \( \beta_1 \geq 0, \) the AR(1)-TBL-GARCH(1, 1) regression model is expressed as

Mean equation: \[ Y_t = a + b \times X_t + \epsilon_t, \quad \epsilon_t = \phi_1 \times \epsilon_{t-1}. \]

\[ \epsilon_{t-1} = \sqrt{h_{t-1}} \epsilon_{t-1}. \]

where \( \epsilon_{t-1} = \sqrt{h_{t-1}} \epsilon_{t-1}. \)

Variance equation: \[ h_t = a_0 + a_1 \times (\epsilon_{t-1}^2) + a_2 \times (\epsilon_{t-2}^2) + b_1 \times h_{t-1} + \gamma \times \epsilon_{t-1} \times \sqrt{h_{t-1}}. \]

\[ + \beta_1 \times h_{t-1}. \]

(4)

3. Empirical results

3.1. Descriptive statistics

Our dataset contains daily crude oil prices, such as Brent and WTI from January 3, 2013 to October 6, 2014. The two crude oils are considered in this study because they are among the world’s most actively traded commodities. In particular, it is worthwhile to examine the asymmetric conditional volatility and asymmetric tail dependence between two return series in that these oil futures contracts show high price fluctuations in recent years. We let \( r_t \) be the compound return of an observed daily crude oil price process in discrete time, \( t = 1, 2, \ldots, n. \) Table 1 presents the descriptive statics of our sample dataset. All return series of Brent and WTI are close to zero mean. Brent and WTI are slightly skewed to the right relative to the normal distribution and the kurtoses of all return series of Brent and WTI are higher than three. Their excess kurtoses are significantly positive, indicating that they have heavy tails relative to the normal distribution.

We inspect the Autocorrelation Function (ACF) and Partial ACF (PACF) of the residual series. The ACF and PACF plots in Fig. 1 show that there is no significant autocorrelation left in the residuals. We also employ a formal method such as the McLeod-Li test in order to test serial correlations and the volatility clustering effects of Brent oil price returns. The figure displays that the \( p \)-values in most lags, except for at 10, 11, 12, and 14 lags, are smaller than the 5% significance level.

Note: This table reports the asymmetry of tail dependence between WTI and Brent crude oil returns by using a broad class of copula functions. The Normal copula has no tail dependence, the Clayton copula has lower tail dependence, the Rotated Clayton copula has upper tail dependence, the Plackett copula has symmetric tail dependence, the Frank copula has symmetric tail dependence, the Gumbel copula has upper tail dependence, the Rotated Gumbel copula has lower tail dependence, the Student-t copula has symmetric tail dependence, and the SJC copula allows for asymmetric tail dependence. A value is highlighted in the statistics for asymmetric tail dependence. Sample period: January 3, 2013 to October 6, 2014.

Therefore, we are able to reject the null hypothesis of homoscedasticity and that the Brent return series have the Autoregressive Conditional Heteroskedastic (ARCH) effect, which is in fact, typically observed in the financial time series. In Fig. 2, it is obvious that there exist weak ARCH effects at the 5% significance level at 1 and beyond 21 lags (long-term lags) in the return series. From visual inspection, we find the existence of conditional heteroskedasticity in the return series of the crude oil prices. Therefore, we consider the Generalized ARCH (GARCH) model (Bollerslev, 1986), in order to eliminate the serial dependence of Brent and WTI. In Section 3.3, we will use the asymmetric GARCH regression models to investigate the asymmetric conditional volatility.

To select the dependent and independent variables for our empirical analysis in the next section, we examine the relationship between Brent and WTI. An obvious implication is that shocks on a specific market quickly affect the other crude oil prices. The Granger-Causality Wald test in Table 2 shows that the unidirectional relationship from Brent to WTI is not statistically significant.
Brent to WTI is statistically significant at the 1% significance level. This indicates that Brent oil prices do Granger cause WTI oil prices. However, WTI oil prices do not Granger cause Brent oil prices. The empirical test provides evidence that information on previous Brent returns plays a role in explaining future returns of WTI over our sample period. Based on the Granger-Causality test results, we will consider WTI as a response variable and Brent as an explanatory variable in the following asymmetric conditional volatility analysis.

We also test whether the oil markets are cointegrated. Stock and Watson (1988) observe that every time series in a cointegrated set can be expressed in terms of common stochastic trends. We use the statistics that Stock and Watson (1988) propose for common trends testing. Table 3 shows that the first column contains the null hypothesis that the k-dimensional time series has m common stochastic trends; the alternative used for the test in the second column indicates that the time series has s common trends, where s < m. The test statistic for testing 2 versus 1 common trends (-218.54) is less than the critical value (-23.00). Therefore, the test rejects the null hypothesis, which means that the series has a single common trend. Thus, Brent and WTI return series are cointegrated with a single common trend. For further investigation, we will use the tail dependence by employing various copula functions in the next subsection. The Granger-Causality Wald test and Stock-Watson’s common trend test yield evidence that Brent and WTI oil prices have a single common trend along with the fact that the Brent oil returns have more influence on the WTI oil returns.

3.2. Asymmetry of tail dependence by copula

We employ the most popular copula functions to investigate the tail dependence between WTI and Brent oil market returns during our sample period. Our goal in this analysis is to examine the asymmetry of tail dependence by using copula models. As we discussed earlier, the return series of Brent and WTI are not normally distributed. The finding is a good reason for us to use a copula approach in order to examine the structural dependence between Brent and WTI oil prices. For a marginal distribution model, we consider the nonlinear-asymmetric GARCH (NAGARCH) of Engle and Ng (1993), which specifies the volatility \( h_t^2 \) with \( a_0 > 0, a_i \geq 0 \), for \( i = 1, ..., q \), and \( \beta_j \geq 0 \) for \( j = 1, ..., p \) as follows:

\[
\begin{align*}
\gamma_t^2 &= a_0 + \sum_{i=1}^{q} a_i (\gamma_{t-i}^2 - \gamma_{t-i}^2) + \sum_{j=1}^{p} \beta_j h_{t-j}^2.
\end{align*}
\]

In this model, the news impact curve, which measures the possible asymmetric impact of good and bad news at \( t - 1 \) on the conditional variance of \( t \), shifts to the right by \( \gamma_{t-1} \). In particular, by applying a family of NAGARCH models to Brent and WTI oil prices, we are able to avoid the serial dependence in the component time series (Kojadinovic and Yan, 2010).

We generate the vector of standardized residuals from the NAGARCH(1, 1) model, where an error follows Student-\( t \) distribution, and then we use the obtained residuals to various copula models in order to investigate tail dependence between the crude oil prices. With these standardized residuals of Brent and WTI, we carry out this empirical exercise by using Eq. (1). The analysis of tail dependence has been used in the financial time series in order to investigate the behavior of the random variables during extreme events. We examine the probability of the extreme events of an extremely large increase or decrease of both Brent and WTI prices by using tail dependence.

Table 4 presents the lower and the upper tail dependencies of various copula models for oil price returns. There is a symmetric tail dependence between Brent and WTI crude oil prices. The estimated values of \( \rho \) obtained from the best fitting copula model. The average time-varying correlation is 0.59, indicating a strong co-movement between the Brent and WTI crude oil prices. The results of the goodness of fit test are also summarized in the table. They show that the time-varying Normal copula is the best fitting one among the twelve different copula models based on the model selection criteria (Akaike information criterion and Bayesian information criterion).

Fig. 3 displays the time-varying Normal copula correlations, obtained from the best fitting copula model. The average time-varying correlation is 0.59, indicating a strong co-movement between the Brent and WTI crude oil markets. An increase in prices in an oil market leads to an increase in the extreme dependence level of the oil prices in the other market.

3.3. Asymmetric conditional volatility

We estimate the three different asymmetric GARCH regression
J.-M. Kim, H. Jung

Economic Modelling 64 (2017) 409–418

reports that BL-GARCH is the best native asymmetric GARCH specification discussed in Section 2.2. Table 5 presents the estimated impact curves obtained from the different asymmetric GARCH regression models: AR(1)-TARCH(1, 1), AR(1)-BL-GARCH(1, 1), and AR(1)-TBL-GARCH(1, 1), respectively.

Fig. 4. News impact curves for WTI given Brent. Note: This figure illustrates the news impact curves obtained from the different asymmetric GARCH regression models: AR(1)-TARCH(1, 1), AR(1)-BL-GARCH(1, 1), and AR(1)-TBL-GARCH(1, 1), respectively. Sample period: January 3, 2013 to October 6, 2014.

Table 6
Robustness checks: statistics for copulas with Brent and WTI.

<table>
<thead>
<tr>
<th>Copula type</th>
<th>Log-Likelihood</th>
<th>AIC</th>
<th>BIC</th>
<th>$\rho^U$</th>
<th>$\rho^L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>-142.420</td>
<td>-284.837</td>
<td>-284.829</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Clayton</td>
<td>-136.233</td>
<td>-272.462</td>
<td>-272.454</td>
<td>0.588</td>
<td>0</td>
</tr>
<tr>
<td>Rotated Clayton</td>
<td>-99.447</td>
<td>-198.891</td>
<td>-198.883</td>
<td>0</td>
<td>0.512</td>
</tr>
<tr>
<td>Plackett</td>
<td>-145.847</td>
<td>-291.690</td>
<td>-291.682</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Frank</td>
<td>-137.169</td>
<td>-274.334</td>
<td>-274.326</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gumbel</td>
<td>-130.294</td>
<td>-260.584</td>
<td>-260.576</td>
<td>0</td>
<td>0.511</td>
</tr>
<tr>
<td>Rotated Gumbel</td>
<td>-153.959</td>
<td>-307.915</td>
<td>-307.907</td>
<td>0.538</td>
<td>0</td>
</tr>
<tr>
<td>Student’s t</td>
<td>-154.202</td>
<td>-308.396</td>
<td>-308.380</td>
<td>0.319</td>
<td>0.319</td>
</tr>
<tr>
<td>SJC</td>
<td>-153.350</td>
<td>-306.692</td>
<td>-306.676</td>
<td>0.546</td>
<td>0.362</td>
</tr>
<tr>
<td>Time-varying Normal</td>
<td>-145.093</td>
<td>-290.175</td>
<td>-290.151</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time-varying Gumbel</td>
<td>-159.325</td>
<td>-318.639</td>
<td>-318.615</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotated SJC</td>
<td>-158.512</td>
<td>-317.001</td>
<td>-316.953</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: This table reports the asymmetry of tail dependence between WTI and Brent crude oil returns by using a broad class of copula functions. The Normal copula has no tail dependence, the Clayton copula has lower tail dependence, the Rotated Clayton copula has upper tail dependence, the Frank copula has symmetric tail dependence, the Gumbel copula has symmetric tail dependence, the Student’s t copula has asymmetric tail dependence, the SJC copula allows for asymmetric tail dependence. A value is highlighted in the statistics for asymmetric tail dependence. Sample period: October 7, 2014 to November 30, 2016.

is no asymmetric conditional volatility. This result is congruent with our previous findings of no significant difference in the tail dependence at the lower and upper tails.

We explain the relationship between the asymmetry of tail dependence and the asymmetric conditional volatility for the target variable given the auxiliary variable by asymmetric GARCH regression models in this section. Our empirical study shows that these two different approaches consistently provide the convincing and congruent evidence of both no asymmetric tail dependence and no asymmetric conditional volatility. The brief theoretical justification of our empirical result in this paper is that since both copula tail dependence and the asymmetric conditional volatility are calculated by using the same conditional probability distribution function of WTI given on Brent, see Eqs. (1)–(4), we may infer that both the copula tail dependence and the asymmetric conditional volatility share the property of asymmetry.

The news impact curve (NIC) is the functional relationship between conditional variance at time $t$ and the shock term (error term) at time $t-1$. Fig. 4 displays the NICs from the three different asymmetric GARCH regression models in order to show the existence of asymmetric conditional volatility of WTI given on Brent oil prices. The three panels clearly show symmetric NICs, which is consistent with our findings as shown in Table 5. Note that the $p$-values of the test of the asymmetry are not statistically significant in the three asymmetric GARCH specifications. In more detail, the news impact curve obtained from one of our models, the AR(1)-TBL-GARCH (1,1) model, clearly shows that there is no difference between the extent of the positive shock to conditional volatility and the extent of the negative shock to conditional volatility. We conclude that there is no asymmetric conditional volatility of WTI conditional on Brent oil prices. This finding is congruent with the previous findings in our copula tail dependence analysis. Given a large shock to conditional volatility, it describes the same as the market crash and boom probabilities due to the conditional volatility feedback.

3.4. Robustness checks

As a respected reviewer suggested, to ensure the robustness, we conduct a robustness exercise. Our sample period is limited to January 3, 2014 to October 6, 2014 when oil prices are fairly stable hovering
around $100 per barrel. The oil prices dropped significantly after October in 2014. One could point out that our sample period is not a good representation of the oil price behavior. Therefore, we select the recent sample, which covers October 7, 2014 to November 30, 2016 in order to confirm that our findings are still robust. From the new sample dataset, we still confirm that Brent oil prices do Granger cause WTI oil prices by using the Granger-Causality Wald test ($p=0.000$). The causality test also shows that WTI oil prices do Granger cause Brent oil prices ($p=0.003$), which indicates that bi-directional causality between the two oil returns. For the robustness check, we continue to focus on the unidirectional relationship from Brent to WTI oil prices.

We conduct the analysis of tail dependence by using the new sample. Table 6 shows that there is no difference between lower tail dependence and upper tail dependence in the Clayton, Rotated Clayton, Gumbel, and Rotated Gumbel copulas. Note that there is little difference between upper tail dependence and lower tail dependence of symmetricized Joe-Clayton copula. Nevertheless, the finding could not provide firm evidence that there is asymmetric tail dependence between Brent and WTI.

For the T-GARCH model estimation results in Table 7, the value of $\alpha_{11}$ is smaller than that of $\alpha_{12}$. However, the value of $\alpha_{12}$ is not statistically significant at the 5% significance level. We could not find strong evidence that there exists asymmetric conditional volatility for WTI given Brent oil prices. The BL-GARCH model estimation results show that the value of $\gamma_1$ is positive and not statistically significant at the 5% significance level. This finding indicates the existence of asymmetric conditional volatility. The values of $\alpha_{11}$, $\alpha_{12}$, and $\gamma_1$ are not statistically significant at the 5% significance level in the TBL-GARCH estimation. Therefore, we still fail to find strong evidence of the existence of asymmetric conditional volatility. Our asymmetric GARCH regression models for WTI given the Brent oil prices provide consistent findings with our previous results. Our main results appear robust. The two methods for asymmetric tail dependence and asymmetric conditional volatility provide the congruent findings, in that there is no asymmetry of conditional volatility of WTI on Brent and no asymmetry of tail dependence between WTI and Brent returns.

We also illustrate the news impact surfaces by using the new sample. Fig. 5 consistently show that the news impact curve is symmetric for negative and positive news. We note that the each NIC is much wider and bends up more than that shown in our main findings. This could be due to the larger sample sizes.

### 4. Conclusion

In this article, we examine the relationship between the asymmetry of conditional volatility and the asymmetry of tail dependence in the crude oil markets. In particular, our empirical study employs the two different sample datasets, in which each dataset covers the time period of stable and unstable oil prices, individually. We first provide evidence, by using the Granger-Causality test, that previous Brent returns play a role in explaining future returns of WTI over our sample period. In addition, we find that Brent and WTI returns are cointegrated with a single common trend.

After generating standardized residuals of Brent and WTI oil prices, we employ various copula model specifications in order to investigate the tail dependence. We find evidence of symmetric tail dependence between WTI and Brent return series. This result implies that returns across crude oil markets exhibit equal correlation and tend to move together during extreme downturns and upturns in our sample. We also analyze the asymmetric conditional volatility of WTI oil prices conditional on Brent returns by using three asymmetric GARCH regression models. Particularly, in this article, we extend the TBL-GARCH regression model to investigate the existence of an asymmetric conditional volatility of WTI oil prices. The asymmetry of conditional volatility is not found during our sample period.

The empirical findings from the two different approaches are congruent, in that there is no asymmetry of tail dependence and no asymmetry of conditional volatility in the crude oil markets over our two different sample periods. Our study reconfirms the findings of Aboura and Wagner (2016) by showing that asymmetric conditional volatility relates to asymmetric tail dependence.

While Abba Abdullahi et al. (2014) find no positive contemporaneous relationship between trading volumes and returns in oil futures markets by using the GMM model, it could be interesting if one examines the existence of asymmetric conditional volatility given
Abba Abdullahi et al. (2014) study by constructing the conditional marginal distribution of crude oil returns given trading volumes for the WTI and Brent markets, individually, and then make a joint distribution and investigate directional dependence based on various copula functions.

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References


Jeon, J., Oh, Y., Yang, D.Y., 2006. Financial market integration in east asia: regional or
global? Asian Econ. Pap. 5 (1), 73–89.