A quantile-copula approach to dependence between financial assets

Jong-Min Kim, Lucia Tabacu, Hojin Jung

1. Introduction

It is crucial for investors to identify dependence structures for risk management and portfolio diversification. A simple way to measure the dependence is the Pearson correlation coefficient. However, the conventional measure of market dependence might not be a useful indicator. It represents only average correlations without showing the asymmetric correlation in bear and bull market periods. In addition, the Pearson correlation is estimated by assuming a linear relationship between the financial returns, which typically deviate from a Gaussian distribution and exhibit extreme comovement across the markets. Without considering the stylized fact that financial asset returns have fat-tailed return distributions, the linear correlation coefficient can be misleading and provide investors with false market risks.

The purpose of this article is to uncover dependence structures between financial markets: stock, oil, bond, and currency markets. In particular, we consider four financial assets that are not normally distributed: the aggregate stock price index, the West Texas Intermediate (WTI) and Brent crude oil prices, the U.S. interest rates, and the euro exchange rates against the U.S. dollar. Note that we focus only on the dependence structure between the return on U.S. stocks and the individual return on three different assets. In order to correctly estimate dependence, we investigate the correlations across the quantiles and the possible asymmetric tail dependence between markets, using copula models. This study analyzes the linear and nonlinear impacts of these financial assets on the U.S. stock returns. Furthermore, we estimate the effect of asset price movements on the different equity return quantiles.

Our contribution to the empirical literature is twofold. First, this study contributes to the empirical literature showing mixed findings by providing additional evidence on the relationship between the financial assets. Using both linear and nonlinear estimation methods, we find that the stock returns are negatively related to oil prices but positively to U.S. interest rates. Second, we also reconfirm the empirical findings: (1) there is a positive relationship between the dollar value and the S&P 500 index; and (2) there is...
an asymmetric tail dependence between S&P500 and U.S. interest rates. Our key findings can help investors and hedgers make more specific investment strategies.

Sim and Zhou (2015) propose a popular quantile-on-quantile approach that is capable of capturing the effect of the quantiles of crude oil price shocks on the conditional quantiles distributions of U.S. equities. They find that the stock-oil relationship is asymmetric. Reboredo and Ugolini (2016) employ a similar approach to examine quantile and interquantile crude oil price changes on different stock return quantiles before and after the financial crisis in developed economies and in the five BRICS countries. Their main finding is that the impact of upper and lower quantile oil price movements on the same U.S. return quantile was much smaller before the crisis.

Several recent studies examine the relationship between crude oil prices and other commodity or financial asset returns. Baruník, Kocenda, and Vacha (2015) investigate asymmetric volatility spillovers between different petroleum markets before and after the financial crisis. They find that volatility spillovers with negative returns are larger than those with positive ones. However, asymmetry in magnitude decreases after the financial crisis in 2008. Their empirical analysis also reveals that there is no clear directional spillovers among petroleum commodities. Bunn, Chevallier, Le Pen, and Sevi (2017) find the linkage between WTI oil prices and natural gas futures in the U.S., and Hartley and Medlock (2014) reaffirm a cointegration relationship between natural gas prices and oil prices. In particular, Bunn et al. (2017) uncover the underlying factors of the linkage. They find the common factors, such as financial hedging and speculative, explaining the U.S. crude oil and natural gas returns. Hartley and Medlock (2014) find evidence that technological change and exchange rates determine a cointegrating relationship between oil and natural gas through altering their substitutability as well as changing their relative prices in the U.S.

A few empirical works by Arora and Tanner (2013) and Cologni and Manera (2008) find evidence of a significant relationship between oil prices and interest rates. However, there are mixed findings. Cologni and Manera (2008) find that an unexpected oil price shock leads to higher interest rates in Canada, France, and Germany while an increase in oil price has no impact on short-term interest rates in the U.K. In contrast, Frankel (2006) and Akram (2009) find an inverse relationship between oil price and U.S. real interest rates while Alquist, Kilian, and Vigfusson (2013) and Frankel and Rose (2010) do not find a statistically significant inverse relationship between the two variables. Arora and Tanner (2013) show that an unexpected rise in real interest rate is followed by a fall in oil price. According to De Schryder and Peersman (2015), the dollar exchange rate drives the demand for oil in oil-importing countries.

Higher oil prices cause higher production costs that directly affect a firm’s profit margins. However, the impacts of oil price shocks on stock prices depend on industry sectors and the size of oil price shocks. Also, the magnitude of the impacts could be asymmetric between bullish and bearish market periods. There is no consensus on the empirical findings in the literature about the relationships between oil prices and stock market returns. Investment professionals have long been puzzled by the inconsistency of the stock market’s reaction to crude oil price changes. Some strands of empirical studies show an oil price’s negative impact on equity returns (see, e.g. Basher & Sadorsky, 2006; Nandha & Faff, 2008; Kilian & Park, 2009). By contrast, some empirical findings indicate no significant relationship between the two financial asset returns (see, e.g., Apergis & Miller, 2009; Henriques & Sadorsky, 2008; Sukcharoen, Zohrabyan, Leatham, & Wu, 2014). Such effects could be intensified in sectors directly related with the oil price movements.

A number of studies find that oil price changes affect stock market returns. For example, Sadorsky (1999) finds evidence of a significant relationship between crude oil prices and U.S. aggregate stock returns by using the VAR model with GARCH effects. Park and Ratti (2008) find that crude oil price movements have a significant impact on stock returns of European countries and the U.S. between 1986 and 2005. An increase in oil prices is likely to lower stock returns in European countries and the U.S., with the exception of Norway as an oil exporter, where there is a positive relationship between oil price movements and real stock returns. They also find that asymmetric effects of oil price changes occur in the U.S. stock returns and increased volatility of oil prices negatively affect stock returns for most European countries in their sample.

At the disaggregated sector level, Arouri and Nguyen (2010) investigate the relationship between stock returns and oil prices in Europe. Their results show that strong linkage between the two financial assets with considerably different reactions of stock returns to oil price movements across sectors: a positive relationship for oil and gas sectors and a negative relationship for food and beverages. On the other hand, Apergis and Miller (2009) find no linkages between international stock market returns and oil price shocks. In addition, Huang, Masulis, and Stoll (1996) report that the daily price of oil futures has no significant impact on U.S. stock returns.

Interest rates are a critical factor in monetary policy transmission and risk management. Policy makers and portfolio managers are especially interested in understanding the dependence structure between interest rates and stock prices. Interest rates have the potential to affect the stock market through the following channels: firstly, changes in interest rates directly lead to changes in discount rates used in equity valuation, secondly, interest rates movements affect a firm’s cost of financing, thus affect a heavy indebted firm’s future cash flows. These channels imply a negative relationship between the two assets. However, there is a reason that the relationship could be expected to be positive. The two asset returns could move in the same direction due to the flight-to-quality effect in high financial market uncertainty, such as during a global financial crisis. On the one hand, some empirical studies provide a negative linkage between the asset returns (see, e.g., Gan, Lee, Yong, & Zhang, 2006; Sweeney & Warga, 1986). On the other hand, the results of Bulmash and Trivoli (1991) and Geske and Roll (1983) indicate that stock returns are positively related to contemporaneous and lagged year Treasury bill rates.

The relation between exchange rate movements and stock returns has also attracted considerable attention in recent years. A decrease in stock prices lowers investors’ incentives to invest the local stock market and to demand the local currency, and thus leads to currency depreciation. Especially for multinational firms, an increase in local currency value is likely to decrease their profits and
stock prices. Empirical studies, such as Bartov and Bodnar (1994) and Jorion (1990), for the U.S. markets fail to find a significant linkage between U.S. firms’ stock returns and U.S. dollar movements. Aggarwal (2003) finds a positive correlation between the trade-weighted dollar and U.S. stock prices while Soenen and Hennigar (1988) show that U.S. stock indexes are negatively correlated with a fifteen currency-weighted value of the dollar. Although, as aforementioned, the bulk of the empirical research has investigated the comovement and causality of the asset returns considered in this study, the lack of unequivocal evidence on the relationship deserves particular attention.

The rest of the paper is organized as follows. The next section introduces the econometric methods, such as a quantile regression and quantile-copulas. Section 3 reports and discusses the empirical results. Summary conclusions are provided in Section 4.

2. Econometric methodology

2.1. Quantile regression

A quantile regression method is an extension of the classical regression that offers information on the whole conditional distribution of the response variable. While in the classical regression case, the goal could be to approximate the conditional mean, in quantile regression, the focus could be to approximate the conditional quantile functions of a response variable $Y$ given a set of variables $X$. The quantile regression model can capture the information associated with the location, scale, and the shape shift of the conditional distribution. The method is especially useful when heteroskedasticity is involved and when the usual parametric assumptions do not hold in homogenous regression models. It is also well known that no error distribution is imposed in quantile regression. We start with an equivalent definition of the $\theta$-quantiles (see Davino, Furno, & Vistocco, 2013).

$$q_\theta = \arg\min_c E[\rho_\theta(Y - c)],$$

where

$$q_\theta(y) = [(1 - \theta)\mathbf{1}(y \leq 0) + \theta\mathbf{1}(y > 0)]y_l$$

denotes an asymmetric absolute loss function. The conditional quantile is defined as

$$\hat{q}_\theta(\theta, X) = \arg\min_{\theta, \beta} E[\rho_\theta(Y - \theta(X))],$$

where $Q_\theta(\theta, X) = Q_\theta(Y|X = x)$ denotes the conditional quantile function for the $\theta$-quantile. For the linear model case $Q_\theta(Y|X) = X\hat{\beta}(\theta)$ we have

$$\hat{\beta}(\theta) = \arg\min_{\theta, \beta} E[\rho_\theta(Y - X\hat{\beta})]$$

where the parameters and the estimators correspond to the specific quantile $\theta$. In the quantile regression linear model, the parameter estimates have the same interpretation as in the classical linear models. The coefficient

$$\hat{\beta}(\theta) = \frac{\partial Q_\theta(Y|X)}{\partial x_i}$$

indicates the change rate of the $\theta$-quantile in the dependent variable distribution per unit change of the $i$-th independent variable. Quantile regression estimators have the same equivariance property as the ordinary least square (OLS) estimators but the equivariance to monotone transformations is specific only to quantile regression (see Koenker, 2005 for more details).

2.2. Quantile-copulas

A copula $C$ is a multivariate distribution with uniform marginals over (0,1). If $U$ is a $p$-dimensional random vector on the unit cube, then the copula $C$ is expressed as

$$C(U_1, ..., U_p) = P(U_1 \leq u_1, ..., U_p \leq u_p).$$

Sklar (1959) shows that there exists a $p$-dimensional copula $C$ such that

$$C(F_1(x_1), ..., F_p(x_p)) = F(x_1, ..., x_p),$$

where $F$ is a $p$-dimensional distribution function with marginals $F_1, ..., F_p$ and for all $x$ in the domain of $F$. Copulas can separate marginals and dependence structures, which is very useful for finding different impacts of upper and lower quantiles.

A $n$-dimensional copula is a function of the following distribution, $C: [0, 1]^n \to [0, 1]$, whose domain is the entire unit square with the three properties: (1) $C(u_1, ..., u_{i-1}, 0, u_{i+1}, ..., u_p) = C(0, v) = 0$, the copula has a zero value if at least one of the arguments is zero, (2) $C(1, ..., u, 1, ..., 1) = u$ if all arguments are 1 with except for one argument of $u$, and (3) $C$ is $n$-increasing such that the $C$-volume of $B = X_{i=1}^n [x_i, y_i] \subseteq [0, 1]^n$ is non-negative. That is, $\int dC(u) = \sum_{x \in \mathcal{X}^n_{i=1}[x_i,y_i]} (-1)^{N(z)} C(z) \geq 0$, where the $N(z) = \#\{k: z_k = x_k\}$ and $\#$ indicates the cardinality of the number of elements of the set.

Tail dependence is the probability of comoving among random variables at the tails for a multivariate distribution. Nelsen (2006) defines the coefficient estimates of upper and lower tail dependence as follows
dependence by using Kendall’s \( \tau \) for continuous bivariate variables is measured as given
\[
\tau_c = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1.
\]

Copulas are used in quantile regression to understand the full joint distribution. We follow Nelsen (2006) to present the algorithm for copula quantile regression. Let \( X \) and \( Y \) be continuous random variables with joint distribution \( F \) with their marginal distribution functions \( F_X \) and \( F_Y \) and copula \( C \). Then, with joint distribution \( C, U = F_X(X) \) and \( V = F_Y(Y) \) are uniform \((0,1)\) random variables. The \( p \)-quantile regression curve of \( Y \) on \( X \), denoted by \( y = \hat{f}(x) \), is the solution to the equation \( P(Y \leq y | X = x) = p \) for \( x \) in the range of \( X \). Then we have
\[
P(Y \leq y | X = x) = P(V \leq \hat{F}_Y(y) | U = F_X(X) = x) = \frac{\partial C(u, v)}{\partial u}
\]

We consider \( \frac{\partial C(u, v)}{\partial u} \) as the partial derivative of the copula function at \( (u, v) \) with respect to \( u \). For every \( u \in [0, 1] \), the partial derivative \( \frac{\partial C(u, v)}{\partial u} \) satisfies \( 0 \leq \frac{\partial C(u, v)}{\partial u} \leq 1 \) and the analogous statement also holds such as \( \frac{\partial C(u, v)}{\partial v} \). Also, \( u \rightarrow C_u(u) = \frac{\partial C(u, v)}{\partial u} \) and \( v \rightarrow C_v(v) = \frac{\partial C(u, v)}{\partial v} \) are non-decreasing functions in the domain of \([0, 1]\). As an illustrative example, we use the partial derivative of the Gumbel copula in Table 1 with respect to \( u \) in this study. Then, we define \( C_{\phi, u} \) which is a strictly increasing function of \( v \).
\[
C_{\phi, u}(v) = \frac{\partial C_{\phi, u}(v)}{\partial u} = \exp\left[-\left((-\ln(u))^\phi + (-\ln(v))^\phi\right)^{\frac{1}{\phi}} \times \left((-\ln(u))^\phi + (-\ln(v))^\phi\right)^{\frac{\phi-1}{\phi}}\right] \frac{\phi-1}{\phi} (-\ln(u))^{\phi-1} - \ln(u).
\]

In our empirical analysis, we consider frequently used copula families in the literature, such as an elliptical copula (the Gaussian copula) and Archimedean copulas (the Clayton and Gumbel copulas). In particular, the Gaussian copula can capture symmetric tail dependence while the Clayton (Gumbel) copula captures an asymmetric tail dependence structure, exhibiting greater dependence at the lower (upper) tail than at the upper (lower) tail.

### 3. Data and empirical analysis

The daily dataset used in this study consists of the S&P 500 (S&P500), the WTI (WTI) and Brent (BRENT) crude oil prices, U.S. 10-year Treasury constant maturity rates (TCM), and the Eurozone’s exchange rates against the U.S. Dollar (EUR). All the series in our study are measured as log returns, such that \( n = 100[\ln(p_t) - \ln(p_{t-1})] \). In particular, the exchange rates are from the databases currency exchange rates provided by Professor Werner Antweiler’s website at UBC Sauder School of Business (http://fx.sauder.ubc.ca/data.html). The other variables are obtained from the Economic Research, Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/fred2/).

### Table 2

Descriptive statistics of the log-return series.

<table>
<thead>
<tr>
<th></th>
<th>Obs</th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>ARCH-LM</th>
<th>Ljung-Box(9)</th>
<th>ADF</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR</td>
<td>4,482</td>
<td>0.001</td>
<td>0.005</td>
<td>-4.607</td>
<td>3.033</td>
<td>0.647</td>
<td>-0.126</td>
<td>5.194</td>
<td>236.770***</td>
<td>6.231</td>
<td>-16.147***</td>
</tr>
<tr>
<td>BRENT</td>
<td>4,482</td>
<td>-0.033</td>
<td>-0.037</td>
<td>-18.130</td>
<td>19.890</td>
<td>2.371</td>
<td>0.095</td>
<td>7.598</td>
<td>201.388***</td>
<td>14.658</td>
<td>-15.444***</td>
</tr>
<tr>
<td>WTI</td>
<td>4,482</td>
<td>-0.029</td>
<td>-0.078</td>
<td>-19.144</td>
<td>17.092</td>
<td>2.546</td>
<td>0.104</td>
<td>7.450</td>
<td>507.447***</td>
<td>34.269***</td>
<td>-15.625***</td>
</tr>
<tr>
<td>TCM</td>
<td>4,482</td>
<td>-0.018</td>
<td>0.000</td>
<td>-18.497</td>
<td>9.628</td>
<td>1.949</td>
<td>-0.098</td>
<td>6.886</td>
<td>338.713***</td>
<td>25.625***</td>
<td>-15.539***</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>4,482</td>
<td>0.017</td>
<td>0.054</td>
<td>-9.470</td>
<td>10.424</td>
<td>1.262</td>
<td>-0.233</td>
<td>10.408</td>
<td>1,062.135***</td>
<td>64.777***</td>
<td>-16.931</td>
</tr>
</tbody>
</table>

Note: ARCH-LM represents Engle’s Lagrange multiplier statistic test for autoregressive conditional heteroscedasticity up to 9 lags in our return series. Ljung-Box(9) indicates the Ljung-Box portmanteau test for the serial correlation up to 9 lags. ADF test shows the augmented Dickey-Fuller test statistics up to 16 lags. The null hypotheses are that there is no ARCH effect, no serial correlation, and nonstationarity, respectively. The values are expressed as percentages. *** denotes statistical significance at the 1%.
Table 2 presents the descriptive statistics for the daily return series. The table clearly exhibits that all of the variables have roughly zero mean values. The daily mean of the U.S. stock returns is 0.017%, which is an annualized return of 6.025%. S&P 500 is slightly skewed to the left compared to the normal distribution, which implies that the stock index has decreased during the sample period. The notable feature of our return series is high kurtosis. The kurtosis values are greater than three, which is typically considered as a threshold for a normal distribution. The high kurtosis values indicate heavy tails of the sample return series. The reported statistics indicate that in general our log return series have stationarity, but our log return series also have serial correlation and ARCH effects. Thus, the test statistics strongly suggest that we need to obtain serial independent standardized residuals before conducting copula analysis to accurately capture the dependence structure.

3.1. Linear dependence

The strength of dependence varies across return series as shown in Table 3; it is below 0.1 in absolute values for the pairs of EUR-TCM and EUR-S&P500(minimum 0.030), and for the remaining returns, it is above 0.1 (maximum 0.593 for BRENT-WTI). We also find that the parametric correlation coefficients are stronger than nonparametric ones, such as Kendall’s τ and Spearman’s ρ, except for EUR-TCM. U.S. interest rates and stock market returns move together while there is a negative relationship between the other return series and S&P500 during the sample period. Note that the correlation coefficient estimates for all pairs considered in this study are statistically significant at the 5% significance level, except for the EUR-S&P500 pair by Kendall’s τ and Spearman’s ρ. From the finding that there exists a dependence across the financial markets, we can infer that both the U.S. monetary policy and oil policy are likely to affect the U.S. stock market returns and any shocks in the oil and bond markets can be transmitted to the stock market. Lastly, we observe that there is a positive relationship between U.S. interest rates and stock prices. When U.S. interest rates rise, U.S. bonds become comparatively more attractive for speculative investors, and U.S. stocks also become more attractive during the sample period.

We also examine the dependence structure between the U.S. stock market and influential factors by using the quantile regression approach. The correlation coefficients in Table 3 provide no information on upper or lower tail dependence between various classes of assets. In contrast, this quantile regression approach clearly shows the link at various quantiles. Table 4 reports the impacts of various shocks from exchange rates, oil prices, and U.S. interest rates on the distribution of U.S. equities. We find empirical evidence that the U.S. stock returns are tightly related with WTI oil prices. The effects appear to be strong, particularly at large, positive or negative WTI oil price shocks. We also observe that Brent oil price shocks on the upper U.S. stock return quantiles (for example, 0.9 quantile) are statistically significant.

As shown in Table 4, the U.S. stock market exhibits an asymmetric dependence with BRENT while WTI displays a symmetric tail
Table 4
Quantile regression results for financial assets.

<table>
<thead>
<tr>
<th>Quantiles</th>
<th>Mean (.1)</th>
<th>(.2)</th>
<th>(.3)</th>
<th>(.4)</th>
<th>(.5)</th>
<th>(.6)</th>
<th>(.7)</th>
<th>(.8)</th>
<th>(.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.020</td>
<td>−1.314***</td>
<td>−0.706***</td>
<td>−0.402***</td>
<td>−0.151***</td>
<td>0.046***</td>
<td>0.254***</td>
<td>0.463***</td>
<td>0.741***</td>
</tr>
<tr>
<td>(0.017)</td>
<td>(0.043)</td>
<td>(0.024)</td>
<td>(0.018)</td>
<td>(0.017)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.023)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>EUR</td>
<td>−0.080**</td>
<td>−0.017</td>
<td>−0.099**</td>
<td>−0.096**</td>
<td>−0.080**</td>
<td>−0.072**</td>
<td>−0.068**</td>
<td>−0.088**</td>
<td>−0.063</td>
</tr>
<tr>
<td>(0.028)</td>
<td>(0.053)</td>
<td>(0.036)</td>
<td>(0.026)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.024)</td>
<td>(0.031)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>BRENT</td>
<td>0.012</td>
<td>0.000</td>
<td>0.011</td>
<td>0.007</td>
<td>0.003</td>
<td>0.004</td>
<td>0.008</td>
<td>0.018*</td>
<td>0.038**</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.016)</td>
<td>(0.012)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.011)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>WTI</td>
<td>−0.066***</td>
<td>−0.074***</td>
<td>−0.059***</td>
<td>−0.051***</td>
<td>−0.047***</td>
<td>−0.046***</td>
<td>−0.049***</td>
<td>−0.050***</td>
<td>−0.053***</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.016)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>TCM</td>
<td>0.217***</td>
<td>0.243***</td>
<td>0.223***</td>
<td>0.195***</td>
<td>0.178***</td>
<td>0.166***</td>
<td>0.172***</td>
<td>0.176***</td>
<td>0.188***</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.016)</td>
<td>(0.012)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.018)</td>
</tr>
</tbody>
</table>

Note: ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. Standard errors are in parentheses.

independence with the market, but the degree of comovements shows a clear U-shape. Stronger dependence is observed between the stock returns and exchange rates. Moreover, they are independent at the both tails and dependent for the intermediate quantiles. The effect of TCM is positive and significant across quantiles with an intensity of the comovements being different. Thus, the structure of dependence is asymmetric, with relatively lower dependence in the intermediate quantiles. When the stock market is neither bearish nor bullish (i.e. at the middle of U.S. return quantiles), the U.S. interest rates have little effect on the U.S. returns. This finding indicates the larger effect of the U.S. monetary policy on U.S. equity returns during extreme periods of market performance.

Fig. 1 is a summary of the quantile regression results, which provides strong evidence of asymmetric tail dependence across the financial markets in our sample. Each plot corresponds to one coefficient in the quantile regression model. Each black dot represents the estimated coefficients for the quantiles indicated on the X axis with the shaded gray area indicating a 90% confidence interval. The red lines represent the OLS estimate of the mean effect and its 90% confidence interval. Notice that in the subfigures like WTI and TCM the quantiles are beyond the least squares estimate. In other plots, the linear regression is sufficient for describing the relationship between the dependent variable and the independent variables. Thus, the quantile estimates are not statistically different from the least squares estimate during the sample period.

3.2. Non-linear dependence

Functional data analysis (FDA), which creates functional data from discrete observations, has recently begun to receive attention in the financial market analysis. In particular, functional principal component analysis (FPCA) is one of the popular tools for the empirical analysis with financial data. It is capable of capturing the directions of variation and reduce dimensions of data. Besides the extensive discussion on FPCA in Ramsay and Silverman (2005), other relevant studies include Reiss and Ogden (2007) and Silverman (1996).

The biplot in Fig. 2 shows the principal component scores (the black cloud) and the loading vectors (the red arrows) corresponding to the first two principal components. We notice that the first loading vector gives approximately positive equal weight to TCM and S&P 500 and the negative weights to the other variables. WTI also has the lowest weight on the first loading. The variables are clustered according to their loading weights on the first principal component. The second loading vector has most of its weight on WTI.

In the time series regression analysis, it is common for errors (residuals) to have non-constant variance. If the residuals have such time series structure, the OLS regression should not be considered because the basic assumption of independent errors are violated. Otherwise, the coefficient estimates and their standard deviations from OLS will be misleading. Note that we are able to adjust when the residuals have an ARMA structure. A linear correlation coefficient cannot be an appropriate approach to measure dependence between non-normal return series (Embrechts, McNeil, & Straumann, 2002). Instead, a nonparametric correlation will be considered as an alternative. As we discussed in the summary statistics, our return series shows non-normal distribution with heavy tails and skewness.

Given such empirical evidence, we utilize the Gaussian copula marginal regression (GCMR) and quantile copulas in order to characterize a dependence structure between the return series. According to Uhlm, Kim, and Jung (2012), one of the most powerful advantages of using copula functions is that they do not require a normal distribution or independent and identical distributed random variables (iid) assumptions. Thus, we are able to consider a Gaussian copula function even when the time series are clearly non-normal. Our choice can be further supported by Guolo et al. (2014), Kim and Hwang (2017), and Masarotto and Varin (2017), showing that the serial dependence and variable dispersion can be modeled by the Gaussian copula. The gcmr R package and a likelihood approach are used to conduct inference. The parameter is estimated by using the maximum likelihood estimation (MLE) method based on 4,482 observations. Our study considers four various linear models incorporating the error correlation structure ARMA(p,q), where p = 0, 1 and q = 0, 1. The best-fit model to the sample is ARMA(1,1) based on the Akaike information criterion

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1 Oswald, Nisbet, Chiaradia, and Arnold (2012) mention that a parametric method with small samples is preferred due to its unbiased estimator with high variance and computation efficiency. The estimator from a nonparametric method will be biased with small sample size when we have a large dataset (about more than 10,000 observations). However, its estimates are not reliable with smaller samples. Therefore, in spite of the flexibility in the nonparametric method, we consider only parametric copula because our sample size is relatively small in this study.
Therefore, our study considers the GCMR with ARMA(1,1) error structure, which provides investors or policymakers with more reliable estimates than the other models.

Table 5 presents our estimates, indicating dependence parameters across the financial markets. There exists a negative relationship between S&P500 and EUR at the 10% significance level. This implies that an appreciation of the U.S. dollar is associated with an increase in the returns of the U.S. stock market. In other words, a stronger dollar leads to higher stock prices in the U.S. Price changes of crude oils are frequently considered as a key factor for fluctuations in stock prices. Oil prices are likely to affect a firm’s costs of factor inputs. Negative oil price impacts could be greatest for companies significantly devoted to oil-based inputs while the extent will be partially dependent on their abilities of hedging oil price shocks. Consistent with our expectation, the estimated

Fig. 1. Plot of quantile regressions.
coefficients are negatively significant for WTI, but not statistically significant for BRENT. WTI oil price increases are associated with the U.S. stock market declines, but the changes of U.S. stock prices during our sample period cannot be explained by the movements in the Brent oil prices. The price increase of WTI could be an important factor in driving down the stock market returns during the sample period.

Table 6 presents the values of the lower ($r^L$) and upper ($r^U$) tail dependence coefficients between S&P 500 and other return series. The coefficients of upper (right) are higher than lower (left) tail dependence for the considered pairs with the exception of EUR-S&P 500. The first glance of the result is that there are weak extreme market comovements, reaching a maximum of 0.276 for TCM-S&P 500. This asymmetric dependence implies that dependence across those markets is stronger during the boom than the bust periods. Also, we notice that there is no asymmetric dependence between foreign exchange and stock markets. This finding provides a useful way to hedge risk and suggests that investors hold portfolio diversification.

Table 5
Empirical results of GCMR with different ARMA (p,q) specifications.

<table>
<thead>
<tr>
<th>Model</th>
<th>ARMA(0,0)</th>
<th>ARMA(0,1)</th>
<th>ARMA(1,0)</th>
<th>ARMA(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.412***</td>
<td>0.413***</td>
<td>0.413***</td>
<td>0.413***</td>
</tr>
<tr>
<td>EUR</td>
<td>−0.023</td>
<td>−0.024*</td>
<td>−0.024*</td>
<td>−0.024*</td>
</tr>
<tr>
<td>BREN</td>
<td>0.010</td>
<td>0.009</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>WTI</td>
<td>−0.113***</td>
<td>−0.113***</td>
<td>−0.113***</td>
<td>−0.111***</td>
</tr>
<tr>
<td>TCM</td>
<td>0.302***</td>
<td>0.300***</td>
<td>0.301***</td>
<td>0.300***</td>
</tr>
<tr>
<td>AIC</td>
<td>1056.600</td>
<td>1047.800</td>
<td>1048.600</td>
<td>1041.800</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>522.280</td>
<td>516.910</td>
<td>517.290</td>
<td>512.920</td>
</tr>
</tbody>
</table>

Note: ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. Standard errors are in parentheses.
Finally, we implement the copula method proposed by Nelsen (2006) for fitting a nonlinear quantile regression model. Fig. 3 also displays the confidence intervals for the fitted quantiles using a Monte-Carlo method. As we discussed in Table 3, the estimated Kendall’s $\tau$ correlation coefficients are close to or less than zero in our sample data. In the Clayton copula function, the parameter is restricted as $\theta \in (0, \infty)$. However, in our sample, it appears to be negative for the pairs; BRENT-S&$P500$ and WTI-S&$P500$. The fitted quantiles for those pairs with the Gumbel copula function display horizontal lines near zero correlation coefficients. We also notice that EUR-S&$P500$ shows the similar shape by using both copula functions considered. Therefore, in this study, we only report the fitted quantiles for the pair of TCM-S&$P500$ in Fig. 3.

Fig. 3 exhibits graphical evidence that interest rate-stock return dependence dramatically changes for upward and downward stock return quantiles. We capture the quantile dependence structure through marginal models obtained by copulas. The figure presents that the right-tail dependence is stronger than the left-tail dependence between bond and stock markets in the U.S. This observation suggests that the U.S. stock returns react asymmetrically to different signs and sizes of the U.S. interest shocks. In particular, the impact of high U.S. interest rates shocks on the upper U.S. equity return quantiles is larger than that of low U.S. interest rates shock quantiles on the lower quantiles of the U.S. stock return distribution. This new evidence of asymmetric quantile dependence structure between bond and stock markets suggests that investors should adapt different portfolio risk management during upturns and downturns in bond markets.

4. Conclusion

In this article, we investigate the linkages between the U.S. stock prices and other financial assets, such as oil prices, exchange rates, and the U.S. interest rates. In particular, we investigate the impacts of the upper and lower quantile movements of various financial asset prices on different equity return quantiles. Our analysis is mainly based on various linear and nonlinear estimation methods, such as correlation coefficients, quantile regressions, GCMR, and quantile-copulas. Our results consistently show strong significant links for the considered pairs of WTI-S&$P500$ and TCM-S&$P500$. Also, we provide strong evidence that there is asymmetric tail dependence between S&$P500$ and the U.S. interest rates. Such results add further evidence to the mixed findings in the empirical literature.

Our evidence that extreme upward and downward interest rate changes have an asymmetric impact on the U.S. stock returns, which helps investors with investment decisions about downside or upside portfolio risk management. For example, investing in the U.S. stock market, when oil price is expected to increase, could be a profitable investment strategy. Investors and portfolio managers should consider risk diversification opportunities across markets. Lastly, the U.S. policy makers should consider not only stock price fluctuations but also interest changes, given that policy effectiveness arise in joint movement between their stock and bond markets.

In our empirical analysis, the dependence coefficient estimates at the upper tail are higher than those at the lower tail, which indicates that the considered financial assets comove during bullish times. With the finding of low correlations and the existence of
Our findings provide useful implications for the diversification sought by investors and policymakers. However, our study considers only the dependence structure between the return on U.S. stocks and each return on three different assets in isolation of one another. To overcome this limitation, future research may focus on the multivariate dependence structure by adopting Almeida, Czado, and Manner’s (2016) approach, which allows the existence of simultaneous interactions across all possible return pairs. In particular, our model is capable of capturing the high-dimensional multivariate dependence structure by extending the stochastic autoregressive copula model. One can also examine the dynamics of the dependence structure between financial assets by using Kim and Hwang’s (2017) methodology for directional dependence by copula.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.najef.2019.101066.

References


