Dynamic functional connectivity analysis of functional MRI based on copula time-varying correlation

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\textbf{GRAPHICAL ABSTRACT}

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|c|c|}
\hline
ROI Pair & Model & 3yo & 4yo & 5yo & 7yo & 8-12yo & Adult \\
\hline
(RTPJ, LTPJ) & GCTV & 140 & 115 & 161 & 160 & 139 & 159 \\
 & TCTV & 136 & 126 & 156 & 153 & 162 & \\
 & DCC & 105 & 123 & 146 & 147 & 123 & 157 \\
(RTPJ, dPCC) & GCTV & 112 & 99 & 160 & 138 & 138 & 134 \\
 & TCTV & 94 & 121 & 154 & 136 & 142 & 136 \\
 & DCC & 72 & 71 & 152 & 118 & 141 & 118 \\
(RTPJ, dACC) & GCTV & 16 & 16 & 91 & 90 & 39 & 39 \\
 & TCTV & 40 & 12 & 97 & 55 & 49 & 38 \\
 & DCC & 18 & 27 & 74 & 45 & 36 & 17 \\
(RTPJ, V1) & GCTV & 11 & 15 & 73 & 33 & 26 & 65 \\
 & TCTV & 12 & 11 & 81 & 24 & 36 & 70 \\
 & DCC & 10 & 9 & 45 & 23 & 18 & 45 \\
(LTPJ, dPCC) & GCTV & 158 & 156 & 163 & 155 & 148 & 161 \\
 & TCTV & 158 & 156 & 163 & 161 & 151 & 158 \\
 & DCC & 154 & 129 & 161 & 149 & 152 & 158 \\
(LTPJ, dACC) & GCTV & 16 & 14 & 65 & 23 & 27 & 75 \\
 & TCTV & 14 & 18 & 68 & 30 & 39 & 63 \\
 & DCC & 7 & 19 & 50 & 37 & 26 & 47 \\
(LTPJ, V1) & GCTV & 31 & 20 & 85 & 56 & 64 & 87 \\
 & TCTV & 18 & 31 & 83 & 52 & 54 & 95 \\
 & DCC & 21 & 17 & 55 & 41 & 46 & 55 \\
(dPCC, dACC) & GCTV & 12 & 3 & 34 & 38 & 31 & 65 \\
 & TCTV & 13 & 6 & 27 & 32 & 37 & 57 \\
 & DCC & 15 & 9 & 38 & 32 & 23 & 32 \\
(dPCC, V1) & GCTV & 34 & 25 & 68 & 91 & 64 & 88 \\
 & TCTV & 35 & 16 & 61 & 85 & 76 & 110 \\
 & DCC & 33 & 16 & 52 & 70 & 43 & 60 \\
(dACC, V1) & GCTV & 8 & 40 & 53 & 38 & 16 & 72 \\
 & TCTV & 21 & 13 & 83 & 37 & 17 & 58 \\
 & DCC & 11 & 11 & 52 & 49 & 14 & 39 \\
\hline
\end{tabular}
\caption{Nonzero Correlations Times}
\end{table}

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Highlights

- We introduce a measure of dynamic functional connectivity (dFC) for fMRI data.
- The suggested method is effective for data having non-normal distributions.
- Numerical simulation results demonstrated superiority of the proposed method compared to the DCC-GARCH method.
- Group analysis of real fMRI found statistically significant pairs of brain regions.

**ARTICLE INFO**

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- Copula
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- Dynamic functional connectivity
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- Posterior cingulate cortex
- fMRI
- Theory-of-mind (ToM)

**ABSTRACT**

**Background:** Recent studies showed that functional connectivity (FC) in the human brain is not static but can dynamically change across time within time scales of seconds to minutes.

**New Method:** This study introduces a new statistical method called the copula time-varying correlation for dynamic functional connectivity (dFC) analysis from functional magnetic resonance imaging (fMRI) data.

**Results:** Compared to other state-of-the-art statistical measures of dynamic correlation such as the dynamic conditional correlation (DCC), the proposed method can be effectively applied to data having asymmetric or non-normal distributions.

**Comparison with Existing Methods:** Numerical simulations were conducted under various kinds of time-varying correlations and distributions, and it was demonstrated that the proposed method was superior to the DCC-based method for asymmetric and non-normal distributions.

**Conclusions:** fMRI data of 138 human participants watching a Pixar animated movie were analyzed by the proposed method based on five a priori selected brain regions in the cortex. Based on statistical group analysis results, it was discovered that (1) the correlation between the left temporoparietal junction (LTPJ) and the primary visual cortex (V1) and the correlation between the dorsal posterior cingulate cortex (dPCC) and V1 were significantly higher for older age groups (5yo–Adult) more often than for younger age groups (3yo–4yo), and (2) the right temporoparietal junction (RTPJ), LTPJ, and dPCC were significantly correlated in all age groups at most of the scanning time periods.

1. Introduction

Inferring functional connectivity (FC) in the human brain has been raising a lot of attention in recent decades, where the aim is to identify functional interdependency between functionally specialized areas of the brain (Friston, 1994). FC has typically been analyzed by using static measures such as correlation, coherence, and mutual information (Wang et al., 2014). These measures assume that the connection between brain areas is constant during the task periods in a task-based or stimulus-based experimental paradigm.

On the other hand, resting-state (RS) functional magnetic resonance imaging (fMRI) is acquired in the absence of a task or stimulus, and each scanning session takes at least a few minutes. There have been many studies to identify the resting-state networks (RSNs) of functionally synchronized brain regions (Biswal et al., 1995; Lee et al., 2013). In spite of the large number of studies using the static FC measures, however, recent studies showed that FC is not static but can dynamically change across a single RS-fMRI session (Chang and Glover, 2010; Hindriks et al., 2016; Laumann et al., 2017; Liégeois et al., 2017). The dynamic functional connectivity (dFC) refers to FC which may fluctuate over time.

The main purpose of this study is to introduce a new measure of dFC which can be applied to general fMRI time series without any restrictions, such as a specific distribution assumption, stationarity, or a priori selection of sliding window length.

1.1. Background

The research area of dFC has been expanding rapidly over the last few years; see, e.g., Hutchison et al. (2013) and more recently Preti et al. (2017) for extensive reviews on dFC for neuroimaging data analysis.

The sliding window technique is the most basic approach to dFC modeling, where the time series are segmented into a set of time windows of fixed length, and the pairwise correlation is computed on each time window. However, this technique has several shortcomings. For example, window lengths that are too short may lead to spurious fluctuations whereas window lengths that are too long may hinder the detection of true dFCs (Hutchison et al., 2013; Leonardi and Van De Ville, 2015; Lindquist et al., 2014). The window length may be chosen from data, e.g., by detecting change-points (Xu and Lindquist, 2015).

Alternatively, model-based approaches have been introduced for measuring dynamic correlations from RS-fMRI (Lindquist et al., 2014; Ryal et al., 2016). The model-based approaches do not need to choose parameters such as the window length in advance, and they have been shown to outperform the sliding-window approach (Lindquist et al., 2014). The model-based approaches for time-varying variances and correlations have been intensively studied in the field of financial time series analysis (Tsay, 2005). Since the autoregressive conditional heteroscedastic (ARCH) model and the generalized ARCH (GARCH) model were proposed for forecasting time-varying variances in Engle (1982) and Bollerslev (1986), the family of GARCH models has been widely developed; see, e.g., Bauwens et al. (2006) and Engle (2004) for systematic overviews. The constant conditional correlation (CCC) GARCH model is a multivariate GARCH model proposed by Bollerslev (1990). The CCC-GARCH is a computationally simple model which produces positive definite conditional covariance matrices, but the conditional correlations are assumed constant over time. The dynamic conditional correlation (DCC) GARCH model was suggested by Engle (2002) by extending the CCC-GARCH model, where the conditional correlations are allowed to be time-varying. The DCC-GARCH was applied for dynamic correlation analysis of RS-fMRI data by Lindquist et al. (2014).

On the other hand, the model-based approaches are not free from distribution assumptions. A copula is a multivariate function that can describe any multivariate distributions by combining their marginal distributions (Sklar, 1973). See Nelsen (1999) for an introduction to copulas. Patton (2002, 2006) introduced copulas with time-varying parameters for modeling conditional dependence between exchange rates. Jondeau and Rockinger (2006) proposed a Copula-GARCH model using the skewed Student's t-copula for investigating the interactions between the daily returns of four major stock indices. Bartram et al. (2007) applied the time-varying copula model of Patton (2006) to analyze the dependence between European stock markets. See Manner and Renzikova (2012) for a comparison of different time-varying copula
models.

1.2. Copula-DCC-GARCH for neuroimaging data analysis

In this study, we introduce the Copula-DCC-GARCH model for a distribution-free time-varying correlation analysis of fMRI data. Kim and Jung (2016) developed a linear time-varying regression method based on DCC estimated with copulas for forecasting financial volatility. Kim and Jung (2018) extended the bivariate Gaussian Copula-DCC-GARCH model to multivariate time-varying partial correlation to model the causal relationship between U.S. stock markets. In this study, we empirically compare the Copula-DCC-GARCH models using the Gaussian copula and the Student’s t-copula with the standard DCC-GARCH model based on time series generated under several types of correlation structures and parametric distributions. The simulation results demonstrate that the time-varying correlations estimated by the Copula-DCC-GARCH models are much more accurate than those estimated by the DCC-GARCH model in the cases of non-normal skewed distributions in the sense of mean squared errors (MSEs) and mean absolute errors (MAEs).

We applied the proposed methods to task-free fMRI data obtained from participants watching a silent version of a Pixar animated movie. The fMRI data had been measured and analyzed for a study of the development of brain networks involved in reasoning about others’ minds (Richardson et al., 2018). We show that the fMRI data have skewed non-normal distributions. We applied the proposed method to fMRI time series obtained at five brain regions: the temporoparietal juncture on the right and left hemispheres (RTPJ, LTPJ), dorsal posterior cingulate cortex (dpPCC), dorsal anterior cingulate cortex (dACC), and primary visual cortex (V1), which are brain regions related to cognitive and emotional brain functions or the visual-cognitive task of the experiment. We estimated the pairwise time-varying correlations between these five brain regions based on the suggested method. A statistical group analysis provided clear evidence not only on age-related differences but also on temporal changes in the time-varying correlations between these brain regions.

The remainder of this paper is organized as follows. In Section 2, we describe the suggested Copula-DCC-GARCH model for estimating time-varying correlations between brain regions. Comparative simulation results between copula-based dynamic correlations and standard dynamic correlations are presented in Section 3. The fMRI data preprocessing and statistical analysis results are presented in Section 4. Concluding remarks and discussions are given in Section 5.

2. Copula-DCC-GARCH models for dynamic correlations

2.1. Copulas

A copula is a multivariate distribution function that can effectively describe interdependencies between random variables having arbitrary distributions. Let $X_i$ be a random variable with a marginal cumulative distribution function $F(x_i)$, for $i = 1, 2, \ldots, n$. It has been shown that any joint cumulative distribution function (CDF) $F(x_1, x_2, \ldots, x_n)$ can be rewritten as the marginal CDFs combined by a multivariate function as

$$F(x_1, x_2, \ldots, x_n) = \mathcal{C}(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)),$$

where the function $\mathcal{C}(u_1, u_2, \ldots, u_n)$ is called the copula (Sklar, 1959, 1973). Note that $U_i = F(X_i)$ has a uniform distribution on the interval $[0,1]$. It means that the dependency structure between the random variables is determined by the copula function regardless of the marginal CDF $F_i$. Moreover, any monotone transformation of each random variable does not affect the dependency structure determined by the copula.

For a choice of parametric form for the copula among several types of copulas, we consider elliptic copulas such as the Gaussian copula and Student’s t-copula as described below. The elliptic copulas have convenient properties so that the densities are explicitly expressed in closed forms, and the correlation matrix is incorporated to allow for an explicit parametrization of the time-varying correlations. From Eq. (1), the joint probability density function (PDF) for $F$ can be written as

$$f(x_1, x_2, \ldots, x_n) = c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \prod_{i=1}^n f(x_i),$$

where $c(u_1, u_2, \ldots, u_n) = \partial^2 \mathcal{C}(u_1, u_2, \ldots, u_n)/\partial u_1 \partial u_2 \cdots \partial u_n$ is the PDF for the copula $\mathcal{C}(u_1, u_2, \ldots, u_n)$, and $f(x_i)$ is the marginal PDF. The PDF for the Gaussian copula can be explicitly written as

$$c_{\Phi}(u_1, \ldots, u_n) = \prod_{i=1}^n \Phi(u_i),$$

where $\Phi$ is the standard normal CDF. The PDF for the Student’s t-copula is defined explicitly as

$$c_{\Psi}(u_1, \ldots, u_n) = \prod_{i=1}^n \phi(u_i/\sqrt{1 - \tau}^+),$$

where $\phi$ is the standard normal PDF and $\tau > 0$ represents the degrees of freedom.

2.2. DCC-GARCH model

Multivariate models for temporal dependence in the variances and correlations have been important topics in financial econometrics (Bauwens et al., 2006). In finance, the conditional standard deviation of the underlying asset return is called the volatility. Multivariate modeling of the market return volatility has been applied to asset pricing, portfolio optimization, and risk management.

Before introducing the DCC-GARCH model as a promising tool for modeling time-varying correlations between multiple brain regions in fMRI, the univariate GARCH model should be introduced first. The observed univariate process $x_t$ follows an ARMA(1,1)-GARCH(1,1) model if

$$x_t = \mu + \beta_1 x_{t-1} + \epsilon_t,$$

where $\epsilon_t$ is a sequence of independent random variables with a mean of zero and a variance of one, $\alpha_0 > 0$, $\alpha_1 > 0$, and $\beta_1 < 1$. In Eq. (5), the conditional variance, $\sigma_t^2$, at time $t$ is described as a linear combination of the squared residual process and the conditional variance at the previous time step. The constraint on $\alpha_1 + \beta_1$ implies that the unconditional variance of $\epsilon_t$ is a finite constant while the conditional variance can change over time. $\eta_t$ is often assumed to have a standard normal distribution. An ARMA(1,1)-GARCH($p$, $q$) model can be defined in a similar manner for larger orders $p$, $q > 1$ (Bollerslev, 1986). For typical fMRI data collected at the sampling rate of every 2 or 3 seconds (< 0.5 Hz), larger values of the orders are usually not necessary.

The DCC-GARCH model is one of the most efficient multivariate GARCH models, which gained a lot of popularity recently (Engle, 2002). Suppose that $\epsilon_t = (\epsilon_{t1}, \ldots, \epsilon_{tq})$ is a multivariate time series with a mean of zero and a conditional covariance matrix of $\Sigma_t = (\sigma_t)$. In the DCC-GARCH model, the covariance matrix is separated into the product of the standard deviations and the correlation as

$$\sigma_t = \rho_t \sqrt{\sigma_{t1}} \sqrt{\sigma_{t2}},$$

Let

$$D_t = \text{diag}(\sqrt{\sigma_{t1}}, \ldots, \sqrt{\sigma_{tn}}),$$

$$R_t = (\rho_t).$$

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denote the diagonal matrix of standard deviations and the matrix of correlations. First, each of the conditional variances, i.e., squared standard deviations, is modeled by the standard GARCH(1,1) model as
\[
\sigma_{ii} = \alpha_0 + \alpha_1 \varepsilon_{i,t-1}^2 + \beta_1 \sigma_{i,i-1}^2.
\]
for \(i = 1, \ldots, n\). Second, the correlation matrix is modeled by
\[
\begin{align*}
\mathbf{z}_t & = \mathbf{D}_t^{-1} \mathbf{e}_t, \\
\mathbf{Q}_t & = (1 - a - b) \mathbf{Q} + a \mathbf{z}_{t-1} \mathbf{z}_{t-1}' + b \mathbf{Q}_{t-1}, \\
\mathbf{R}_t & = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2},
\end{align*}
\]
where \(\mathbf{z}_t\) is the standardized residual vector, \(\mathbf{Q}\) is the unconditional covariance matrix of \(\mathbf{z}_0\), and \(a, b > 0\) are the scalars satisfying

![Graphs showing true and estimated correlations](image-url)

**Fig. 1.** True correlation values (constant, red dashed line) and the estimated time-varying correlations (black straight line) by the three methods, GCTV (left column), TCTV (middle column), and DCC (right column). The two independent time series data sets were generated from the normal (top row), Student’s t (second row), log-normal (third row), and beta (bottom row) distributions.
a + b < 1. The covariance matrix \( \Omega_t \) is recursively updated in (9) and then it is normalized to yield the correlation matrix \( R_t \) in (10).

The parameters in the DCC-GARCH model can be estimated by maximizing the conditional likelihood, where the conditional log-likelihood can be decomposed into the sum of the variance term and the correlation term (Ghalanos, 2015). The log-likelihood can be computed by assuming some specific distribution. For instance, in the case of a multivariate normal distribution assumption, each univariate GARCH model can be estimated separately. See Ghalanos (2015) for more details.

2.3. Copula-DCC-GARCH model for time-varying correlation

Note that the parameter estimation for the DCC-GARCH model depends sensitively on the distribution assumption. The copula framework can be joined with the DCC-GARCH model effectively by using elliptic copulas with time-varying correlations to alleviate this problem.

Let \( x_t = (x_{1t}, \ldots, x_{nt})^\text{T} \) denote the vector of the observed fMRI data at time \( t \). The inference using the Copula-DCC-GARCH model is comprised of three steps. One is the estimation of the ARMA(1,1)-GARCH(1,1) model parameters for each marginal distribution, \( F(x_t) \). Two is the computation of the probability integral transform, \( u_t = F(x_t) \), in the copula framework; see, e.g., Eq. (1). Three is the estimation of the joint DCC-GARCH model parameters including the time-varying correlation matrix \( R_t \) in the copula framework using either the Gaussian or Student's t-copula in Eqs. (3) and (4). See A for technical details. Algorithm 1 summarizes the Copula-DCC-GARCH method.

Algorithm 1. The Copula-DCC-GARCH method

Step 1 Estimate the ARMA(1,1)-GARCH(1,1) model parameters in Eq. (5) for each marginal distribution, \( F(x_t) \).

Step 2 Compute the probability integral transform, \( u_t = F(x_t) \), by using the marginal distribution \( F_t \) estimated by fitting the ARMA(1,1)-GARCH(1,1) model.

Step 3 Estimate the correlation matrix, \( R_t \), by maximizing the conditional likelihood in the DCC-GARCH model framework using Eqs. (8), (9), and (10).

Table 1: Mean squared error (MSE) and mean absolute error (MAE) of the estimated correlations for the simulated data using (a) the constant zero correlation, (b) the sinusoidal correlation, or (c) the step correlation. The bold font emphasizes the smallest MSE or MAE values among the three methods, GCTV, TCTV, and DCC.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>GCTV</th>
<th>TCTV</th>
<th>DCC</th>
<th>GCTV</th>
<th>TCTV</th>
<th>DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(0, 1)</td>
<td>0.0018</td>
<td>0.0017</td>
<td>0.0018</td>
<td>0.0184</td>
<td>0.0184</td>
<td>0.0184</td>
</tr>
<tr>
<td>± 0.0022</td>
<td>± 0.0022</td>
<td>± 0.0022</td>
<td>± 0.0022</td>
<td>± 0.0022</td>
<td>± 0.0022</td>
<td>± 0.0022</td>
</tr>
<tr>
<td>t(4)</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0018</td>
</tr>
<tr>
<td>± 0.0015</td>
<td>± 0.0015</td>
<td>± 0.0015</td>
<td>± 0.0015</td>
<td>± 0.0015</td>
<td>± 0.0015</td>
<td>± 0.0015</td>
</tr>
<tr>
<td>Lognormal(0, 1)</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.1288</td>
<td>0.1288</td>
<td>0.1288</td>
<td>0.1288</td>
</tr>
<tr>
<td>± 0.0021</td>
<td>± 0.0021</td>
<td>± 0.0021</td>
<td>± 0.0309</td>
<td>± 0.0309</td>
<td>± 0.0309</td>
<td>± 0.0309</td>
</tr>
<tr>
<td>Beta(5, 2)</td>
<td>0.0022</td>
<td>0.0022</td>
<td>0.8751</td>
<td>0.8751</td>
<td>0.8751</td>
<td>0.8751</td>
</tr>
<tr>
<td>± 0.0025</td>
<td>± 0.0025</td>
<td>± 0.0024</td>
<td>± 0.0395</td>
<td>± 0.0395</td>
<td>± 0.0395</td>
<td>± 0.0395</td>
</tr>
</tbody>
</table>

The joint density function in Eq. (2) is given by either the Gaussian copula (3) or the Student’s t-copula (4).

In this paper, the method using the Gaussian copula for estimating time-varying correlations is called “GCTV”, and the method using the Student’s t-copula is called “TCTV”. In addition, the standard DCC-GARCH method is simply called “DCC” from now on.

In this study, we considered maximum likelihood estimation under the parametric models such as the Gaussian and Student's t-copulas and the ARMA(1,1)-GARCH(1,1) model for marginal distributions. In general, parametric estimation may result in underestimation when the parametric model of the copula or the marginal distributions is misspecified (Charpentier et al., 2007; Kim et al., 2007). Nevertheless, likelihood-based estimators have demonstrated robustness and efficiency against small sample sizes and outliers (Ko and Hjort, 2019; Mendes et al., 2007), and radially symmetric copulas such as the Gaussian and Student’s t-copulas have been shown to be robust to misspecification of copula models (Prokhorov and Schmidt, 2009).

3. Comparative simulations

We compared the suggested copula-based methods, GCTV and TCTV, with the DCC method by using simulated time series data. Bivariate time series, \( y_t = (y_{1t}, y_{2t}) \), \( t = 1, 2, \ldots, 1000 \), were generated representing fMRI measurements from two separate brain regions of interest (ROIs) in the following two steps.

First, \( x_t = (x_{1t}, x_{2t}) \) was generated from the bivariate normal distribution with a mean of zero and a covariance matrix of \( \Sigma_{x_{12}} \), i.e., \( x_t \sim N(0, \Sigma_{x_{12}}) \), where

\[
\Sigma_{x_{12}} = \begin{pmatrix} 1 & \rho_{x_{12}} \\ \rho_{x_{12}} & 1 \end{pmatrix},
\]

for \( t = 1, 2, \ldots, 1000 \). As a result, \( x_t \) is independent between time points while \( x_{1t} \) and \( x_{2t} \) are correlated by a time-varying correlation \( \rho_{x_{12}} \). We
have used three different types of correlation functions for the simulations:

(a) Constant zero correlation: \( \rho_{0t} = 0 \) for \( t = 1, 2, \ldots, 1000 \).
(b) Sinusoidal correlation: \( \rho_{0t} = \sin(2\pi \cdot \frac{t}{200}) \) for \( t = 1, 2, \ldots, 1000 \).
(c) Step correlation: \( \rho_{0t} = 0.5 \) for \( 100k \leq t < 100(k + 1) \), \( k = 0, 2, \ldots, 8 \); \( \rho_{0t} = 0 \) otherwise.

The three correlation functions are depicted in Figs. 1, 2, 3 by red dashed lines. The three correlation functions were selected in consideration of three basic characteristics of neural activities which may arise under various fMRI task conditions. See Section 5 for further discussion.

Fig. 2. True correlation values (sinusoidal, red dashed line) and the estimated time-varying correlations (black straight line) by the three methods, GCTV (left column), TCTV (middle column), and DCC (right column). The two time series data sets were generated from the normal (top row), Student’s t (second row), log-normal (third row), and beta (bottom row) distributions.
of selecting the three correlation functions. Second, each variable $x_i$ was transformed monotonically into one of the four types of distributions:

(i) $N(0, 1)$: The standard normal distribution.
(ii) $t_4$: The Student’s $t$-distribution with $\nu = 4$ degrees of freedom.
(iii) Lognormal(0, 1): The log-normal distribution with location $\mu = 0$ and scale $\sigma = 1$.
(iv) Beta(5,2): The Beta distribution with shapes $\alpha = 5$ and $\beta = 2$.

Let $F(\cdot)$ denote the cumulative distribution function (CDF) of one of the above distributions and $\Phi(\cdot)$ denote the CDF of the standard normal distribution. Fig. 3. True correlation values (step, red dashed line) and the estimated time-varying correlations (black straight line) by the three methods, GCTV (left column), TCTV (middle column), and DCC (right column). The two time series data sets were generated from the normal (top row), Student’s $t$ (second row), log-normal (third row), and beta (bottom row) distributions.
distribution. The transformed data $y_{it}$ can be expressed as

$$y_{it} = F^{-1}(\Phi(x_{it})).$$

In this way, the marginal distribution of $y_{it}$ is converted to have the prescribed distribution. The four distribution functions were selected because they have different characteristics such as skewness and kurtosis. The $N(0, 1)$ and $t_4$ have a skewness of 0, while the Lognormal(0,1) and Beta(5,2) have skewnesses of 0.6185 and -0.596, respectively. The $N(0, 1)$, $t_4$, Lognormal(0,1), and Beta (5,2) have kurtoses of 3, $\infty$, 113.936, and 2.88, respectively.

Fig. 4. Comparison of the mean squared errors (MSEs) of the correlations estimated by the three methods, GCTV, TCTV, and DCC, for the case of constant zero correlation (left column), sinusoidal correlation (middle column), and step correlation (right column). The time series data were generated from the normal ($N(0, 1)$, top row), Student’s $t$ ($t_4$, second row), log-normal (Lognormal(0, 1), third row), and beta (Beta(5, 2), bottom row) distributions.
In order to evaluate the estimation accuracy, we computed the mean squared errors (MSEs) and the mean absolute errors (MAEs) of the estimated time-varying correlations, \( \hat{\rho}_t \), based on 50 repeated simulations as

\[
\text{MSE}(\hat{\rho}_t) = \frac{1}{50(1000)} \sum_{i=1}^{50} \sum_{t=1}^{1000} (\hat{\rho}_t^{(i)} - \rho_{0t})^2, \\
\text{MAE}(\hat{\rho}_t) = \frac{1}{50(1000)} \sum_{i=1}^{50} \sum_{t=1}^{1000} |\hat{\rho}_t^{(i)} - \rho_{0t}|. 
\]  

(13)  

(14)

3.1. Constant zero correlation

Fig. 1 illustrates the estimated time-varying correlations for a sample of bivariate time series data generated under the constant zero correlation. In each panel, true correlation values are depicted as a red dashed line, and the estimated time-varying correlation is depicted as a black straight line. The three methods, GCTV, TCTV, and DCC, were applied while the data were generated based on each of the four types of the distributions. In the figure, it is clearly shown that the DCC (the third column of the panels) fails to estimate the true correlations in the cases of the log-normal distribution and the beta distribution (the third and fourth rows of the panels).

Such observations can be statistically verified based on the MSEs and MAEs obtained from 50 repeated simulations, which are summarized in Table 1(a) together with the standard deviations given as ±SD. The MSE or MAE values of the copula-based methods (GCTV and TCTV) are significantly lower than those of the DCC method in the cases of the log-normal and beta distributions.

The left four panels in Fig. 4 show the boxplots of the MSE values obtained from 50 repeated simulations. We did not include the boxplots of the MAE values because they look similar to the MSE values. We can clearly see that the GCTV and TCTV methods yielded relatively small MSE values for all cases whereas the DCC method produced large MSE and MAE values for the cases of Lognormal(0,1) and Beta(5,2) distributions.

3.2. Sinusoidal correlation

The sinusoidal correlation takes values in a bounded range, \([-1,1]\), and both the negative and positive values are included. It simulates a certain natural situation where the correlation between ROIs may fluctuate over time while the sign of the correlation can also change.

Fig. 2 shows the estimated time-varying correlations for a sample of bivariate time series data generated under the sinusoidal correlation. As in the previous case of the constant zero correlation, the marginal distribution of each variable follows either the normal distribution, Student’s t-distribution, log-normal distribution, or beta distribution, which correspond to the four rows in the figure. In the cases of the normal and Student’s t-distributions, the three methods accurately estimated the time-varying correlations. However, in the case of the log-normal distribution, the DCC method could not estimate negative correlation values, and in the case of the beta distribution, the DCC failed to find the temporal fluctuation of the correlation values.

Table 1(b) summarizes the MSE and MAE values obtained from 50 repeated experiments. In the table, we can compare the performances of the three methods, GCTV, TCTV, and DCC, for estimating the sinusoidal time-varying correlations. It is apparent that the MSEs and MAEs obtained by the DCC are much higher than those obtained by the GCTV and TCTV methods for the cases of the log-normal and beta distributions.

The middle four panels in Fig. 4 illustrate the boxplots of the MSE values obtained by the three methods for the estimation of sinusoidal correlations. The GCTV and TCTV methods consistently produced low MSE values while the DCC produced significantly higher MSE values in the cases of the Lognormal(0,1) and Beta(5,2) distributions.

3.3. Step correlation

The step correlation function, \( \rho_{0s} \), takes the constant value of 0.5 at the repeating intervals, \( t \in [0, 100) \cup [200, 300) \cup \cdots \cup [800, 900) \), and it takes the value of 0 otherwise. This function assumes a certain situation where phase shifts occur in the brain dynamics.

Fig. 3 illustrates the estimated correlations for a sample of bivariate time series data generated under the step correlation. In the figure, we can see that the three methods cannot detect the exact change points, but only up-and-down fluctuations can be correctly detected. All three methods produced relatively accurate estimates of the time-varying correlations for most cases. However, in the case of the log-normal distribution, which corresponds to the third row of the figure, the DCC method could not find the up-and-down fluctuations correctly. In the case of the beta distribution, which corresponds to the bottom row of the figure, the DCC produced correlations which were near 1 over all time points regardless of the actual values of the true correlations.

Table 1(c) presents the MSEs and MAEs together with the corresponding standard deviations obtained by the three methods for estimating the step correlation. Again, the DCC method produced significantly higher MSE and MAE values in the cases of the log-normal and beta distributions. In the other cases, the three methods did not yield significantly different MSE and MAE values.

The right four panels in Fig. 4 illustrate boxplots of the MSE values to compare the three methods for estimating the step correlation. The MSE values of the GCTV and TCTV remained at low values while only the DCC method produced high MSE values for the cases of the Lognormal(0,1) and Beta(5,2) distributions.

4. Statistical analysis of fMRI data

The fMRI data used in this study were obtained from the OpenfMRI database (Poldrack et al., 2013). The accession number of the data is ds000228, and the data are freely available at https://openfmri.org/dataset/ds000228/.

4.1. Participants

The original fMRI data set consists of anatomical and functional MRIs of 155 participants. After the preprocessing steps which will be explained in the next section, we discarded the MRIs of 17 participants. The remaining 138 participants were grouped into six groups according to their ages, which were labeled as ‘3yo’, ‘4yo’, ‘5yo’, ‘7yo’, ‘8-12yo’, and ‘Adult’. Table 2 summarizes the number of participants in each group.

<table>
<thead>
<tr>
<th>Age</th>
<th>3yo</th>
<th>4yo</th>
<th>5yo</th>
<th>7yo</th>
<th>8-12yo</th>
<th>Adult</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
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<td>M</td>
<td>F</td>
<td>M</td>
<td>F</td>
<td>M</td>
<td>F</td>
</tr>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
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<td>6</td>
<td>8</td>
<td>5</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
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<td>7</td>
<td>8</td>
<td>6</td>
<td>16</td>
<td>16</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 2: Number of the participants in each subgroup classified by their age, gender, and handedness.
group, subdivided further by gender and handedness.

We note that the participants have a wide range of ages from 3 to 39 years old. The data were originally collected and analyzed to demonstrate a development of brain networks associated with 'Theory-of-Mind' (ToM) and 'pain' (Richardson et al., 2018), where correlation matrices obtained for each age group have been compared. In this study, we also grouped the participants by their ages in order to verify the age effect on the time-varying correlations.

4.2. Data acquisition and preprocessing

The participants watched a silent version of Disney Pixar’s animated movie ‘Partly Cloudy’, a 5.6-minute short film, during the fMRI data acquisition. The movie was preceded by 10 seconds of rest, and the participants were instructed to remain still while watching the movie.

The MRI data were acquired by using a 3-Tesla Siemens TIM Trio scanner. A total of 168 scans of whole brain images were acquired for

<table>
<thead>
<tr>
<th>Selected ROIs</th>
<th>RTPJ</th>
<th>LTPJ</th>
<th>dPCC</th>
<th>dACC</th>
<th>V1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1st Qu.</td>
<td>0.007</td>
<td>0.011</td>
<td>0.018</td>
<td>0.019</td>
<td>0.016</td>
</tr>
<tr>
<td>Median</td>
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<td>0.085</td>
<td>0.092</td>
<td>0.079</td>
<td>0.119</td>
</tr>
<tr>
<td>Mean</td>
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<td>0.211</td>
<td>0.197</td>
<td>0.172</td>
<td>0.252</td>
</tr>
<tr>
<td>3rd Qu.</td>
<td>0.262</td>
<td>0.355</td>
<td>0.350</td>
<td>0.304</td>
<td>0.444</td>
</tr>
<tr>
<td>Max.</td>
<td>0.993</td>
<td>0.969</td>
<td>0.889</td>
<td>0.850</td>
<td>0.972</td>
</tr>
</tbody>
</table>

4.2. Data acquisition and preprocessing

The participants watched a silent version of Disney Pixar’s animated movie ‘Partly Cloudy’, a 5.6-minute short film, during the fMRI data acquisition. The movie was preceded by 10 seconds of rest, and the participants were instructed to remain still while watching the movie.

The MRI data were acquired by using a 3-Tesla Siemens TIM Trio scanner. A total of 168 scans of whole brain images were acquired for

**Fig. 5.** Sample fMRI BOLD level time series (upper panel) for the two ROIs, LTPJ and dPCC, and the corresponding histograms (lower panels). The sample time series were obtained from a participant in the age group ‘6yo’.
In this study, we selected five ROIs on the cortex in order to investigate dynamic functional connectivity in the brain during the movie watching. Specifically, we selected five Brodmann areas (BAs) (Brodmann, 2006) considering the visual-cognitive task of the experiment and the previous research results which used the same fMRI data.

The five BAs are the temporoparietal junction on the right and left hemispheres (RTPJ, LTPJ), dorsal posterior cingulate cortex (dPCC), dorsal anterior cingulate cortex (dACC), and primary visual cortex (V1). The V1 was selected because of the visual stimulus of the experiment. The other four brain regions, RTPJ, LTPJ, dPCC, and dACC, were selected because the animated movie utilized as the visual stimulus has been shown to activate the Theory-of-Mind (ToM) brain regions including the four brain regions (Jacoby et al., 2016; Richardson et al., 2018). The ToM regions are the brain regions related to thinking about the minds of others during movie-watching (Jacoby et al., 2016). We selected the central locations of the five BAs, whose coordinates were defined in the MNI template by Lacadie et al. (2008). The BA names and the exact MNI coordinates of the selected voxels are listed in Table 3. Then, we took an average of the Blood-Oxygen-Level Dependent (BOLD) signals in the 5 × 5 × 5 cubic region centered at each selected voxel location.

Note that we did not perform spatial smoothing in the preprocessing steps. Instead, we selected cubic regions which are not overlapping each other so that inference on functional connectivity might not be disturbed by erroneous correlations; see, e.g., (Alakörrkö et al., 2017) and references therein.

In addition, we removed the fMRI data of 17 participants whose MRI images had been excessively damaged so that not all of the five brain regions had been included in the original source files. We also removed the first 10 seconds of the scanned images, which correspond to the resting period before watching the movie.

We remark that the selection of ROIs can be changed for future investigation. In this study, we propose a new approach for dynamic functional connectivity analysis, which can be used for exploratory data analysis to discover correlated brain regions from fMRIs. Such a data-oriented approach can provide new insights into brain dynamics which have not been known in previous studies.

4.3. Statistical analysis

Fig. 5 illustrates two sample time series extracted from the fMRI data set at two brain regions, LTPJ and dPCC, of a female participant in the age group ‘4yo’. The two brain regions, LTPJ and dPCC, had relatively high Pearson and Spearman correlation coefficients of 0.679 and 0.663, compared to the participant’s other pairs of brain regions. In order to check the empirical distribution of each sample time series, the histogram is shown for each time series on the lower panel of the figure. It is clear that the marginal distribution for dPCC is quite different from the Gaussian distribution. The Shapiro-Wilk normality test confirmed this observation (p-value = 0.203, 0.001). Both of the marginal distributions do not have fatter tails or outliers due to preprocessing (kurtosis = 2.745, 2.632), but they are skewed (skewness = 0.213, 0.519).

In order to see the results for all the participants, we conducted the Shapiro-Wilk normality test for the fMRI data of every participant. Table 4 shows summary statistics of the p-values obtained for each of the selected ROIs. We can see that the median values are less than 0.1 for all ROIs except V1, and the first quartile values are around 0.01. It means that more than half of the fMRI data have marginal distributions far different from the Gaussian distributions, and more than 25% of the data are significantly different.

Time-varying correlations between the two sample time series illustrated in Fig. 5 were estimated by the three methods, GCTV, TCTV, and DCC, which are shown in Fig. 6. For all methods, we used the ARMA(1,1)-GARCH(1,1) model as the marginal distribution model. In Fig. 6, the three methods successfully identified the fluctuations of the correlation over time. However, the difference between the copula-based time-varying correlations (GCTV or TCTV) and DCC correlations is much larger than that between the GCTV and TCTV, which may have been caused by the non-normality of the fMRI data.

In order to conduct statistical group analyses, the time-varying...
correlations were obtained for all participants and all pairs of the five brain regions. In each age group of the participants, an approximate 99% confidence interval for the mean of the correlation was calculated by

$$\left( \bar{\rho}_g - t_{0.995} \frac{s_g}{\sqrt{n_g}}, \bar{\rho}_g + t_{0.995} \frac{s_g}{\sqrt{n_g}} \right),$$

for each age group $g \in \{‘3yo’,…,’Adult’\}$ and time point $t = 1, 2, \ldots$, where $\bar{\rho}_g$ and $s_g$ are the mean and the standard deviation of the estimated correlation values, $n_g$ is the number of participants in the age group $g$, and $t_{0.995} = T_{n_g-1}(0.995)$ is the critical value of the Student’s $t$-distribution with $(n_g - 1)$ degrees of freedom.

Fig. 7 shows line plots of the mean of the estimated time-varying correlations over the participants in each age group, and the
Table 5  
The number of time points at which the mean of the time-varying correlation is significantly different from zero with the confidence level of 99%. Each cell is colored based on the cell value, i.e., white if the value < 40, light green if 40 ≤ value < 80, light yellow if 80 ≤ value < 120, and light red if 120 ≤ value < 160.

<table>
<thead>
<tr>
<th>ROI Pair</th>
<th>Model</th>
<th>3yo</th>
<th>4yo</th>
<th>5yo</th>
<th>7yo</th>
<th>8-12yo</th>
<th>Adult</th>
</tr>
</thead>
<tbody>
<tr>
<td>(RTPJ, LTPJ)</td>
<td>GCTV</td>
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<td>161</td>
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<td>147</td>
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<td>157</td>
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<td>118</td>
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<tr>
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<tr>
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<td>(dPCC, V1)</td>
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<td>16</td>
<td>52</td>
<td>70</td>
<td>43</td>
<td>60</td>
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<td>(dACC, V1)</td>
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<td>40</td>
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<td>11</td>
<td>52</td>
<td>49</td>
<td>14</td>
<td>39</td>
</tr>
</tbody>
</table>

The correlations were estimated between the two brain regions, dPCC and dACC. The sample mean values, \( \bar{\rho}_{ij} \), are drawn in red straight lines while the upper and lower limits of the 99% confidence interval are drawn in blue dashed lines.

By comparing the lower limit of the confidence interval with the zero value, we can statistically test if the true mean of the correlation is nonzero. That is, if the lower limit is greater than zero, we can conclude that the true mean of the correlation is nonzero with 99% confidence. In Fig. 7, the time points are shaded gray when the lower limit exceeded zero. In the figure, we can see that a relatively large number of time points are colored gray for the older age groups, i.e., '5yo', '7yo', '8-12yo', and 'Adult', whereas most of the time points are not shaded for the younger age groups, '3yo' and '4yo'. We can conclude that age is an important factor for functional connectivity strength and durability between the two brain regions dPCC and dACC.

In order to check the connectivity between all the pairs of brain regions, we counted the number of time points at which the lower limit exceeded zero. Table 5 presents the number of such time points where the true mean of the correlation is nonzero with 99% confidence. Care must be taken when we examine the cell values, however. The numbers of the participants in the groups '5yo' and 'Adult' are relatively larger than the other age groups, which can result in an increase in the cell values, i.e., the null hypothesis is slightly more likely to be rejected. We can see that most of the cell values in the column '5yo' of the table are relatively larger than the cell values in the '4yo' and '7yo' columns.

Based on the results in Table 5, we can discover the following points:

(1) The correlations between LTPJ and V1 and between dPCC and V1 are significantly higher at more time periods for older age groups (5yo–Adult) than for younger age groups (3yo–4yo).

(2) The correlations between RTPJ and LTPJ, between RTPJ and dPCC, and between LTPJ and dPCC are significantly high over all age groups at most of the scanning time periods. It implies that age is not as important of a factor to these connections as to the other connections.
In addition, fluctuation in the estimated time-varying correlations can provide an insight into the characteristic of the connectivity in relation to the visual stimulus. Fig. 8 shows the average time-varying correlation between dPCC and aDCC for the participants in the age group ‘Adult’. In the figure, the mean correlation value suddenly increased and peaked at 204–230 seconds, and then it peaked again at 260–304 seconds. Note that, from the simulation study results in Section 3, fluctuations in the estimated time-varying correlations can indicate either a smooth or sudden change of the correlations. Therefore, estimated time-varying correlations can be used for classification of the changes in stimulus during a movie.

5. Conclusion and discussion

In fMRIs, it is now commonly accepted that the connectivity between ROIs may change over time. In addition to this time-varying nature of the connectivity, fMRI data are often non-normally distributed, which poses challenges to the standard statistical models commonly used in the literature.

In this study, we introduced two copula-based methods, which are called the GCTV and TCTV, for estimation of time-varying correlations between two ROIs from fMRI data. A copula is a multivariate distribution function which can describe dependency between two or more random variables having arbitrary distributions.

In the simulation, we demonstrated that the copula-based proposed methods outperformed the standard DCC-GARCH method in the cases of skewed distributions under various types of time-varying correlation functions. Note that the correlated bivariate data in the simulation were generated based on monotonic transforms of each variable. The results imply that the copula-based methods are competitive for discovering the true time-varying correlations between two ROIs in the brain when the data are corrupted by any non-normal or non-linear monotonic transforms.

Especially, the DCC-GARCH fails to estimate the correlation of time series variables having skewed distributions, such as the log-normal or beta distributions. The skewness of the transform causes a heavy tail in the distribution, which leads to unbalanced outliers in the data. In real situations, such non-normal or non-linear transforms with unbalanced outliers can often be caused by certain systematic errors during the measurement or preprocessing of the data (Murphy et al., 2013; Power et al., 2012).

In the simulation, the data were generated according to three different types of correlation functions, which are the constant zero, sinusoidal, and step correlations. The constant zero correlation represents time-invariant correlations between ROIs throughout the whole fMRI session. The sinusoidal correlation represents smoothly changing correlations, and the step correlation represents non-smooth, abruptly changing correlations. In the sense of experimental paradigms, correlation can change smoothly under task-free conditions whereas abrupt correlation changes can take place under block designs for experiments. In brain imaging, for instance, the onset of epileptic seizures is characterized by synchronization of electroencephalography (EEG) signals (Elger et al., 2006; Jirksa et al., 2013). Especially, step correlation assumes phase shifts in the brain dynamics (Xu and Lindquist, 2015; Cabrieto et al., 2018). In this sense, the simulation study in this work may cover a wide range of fMRI data for research on time-varying correlations between multiple ROIs.

We presented real fMRI data analysis results by using the proposed methods, together with statistical group analysis. The fMRI data used in this study include participants of a wide range of ages, from 3 to 39 years old. The data had originally been collected and analyzed to identify the development of brain networks associated with ‘theory of mind’ (ToM) and ‘pain’ (Richardson et al., 2018).

In this study, we discovered a few pairs of brain regions whose connectivity changed between younger and older age groups. Especially, we could find that the brain region labeled as V1 was involved in those age-sensitive brain networks. It was shown that the correlation between LTPJ and V1 and the correlation between dPCC and V1 were significantly higher during longer time periods for older age groups than for younger age groups. As it has been shown that the ToM networks and pain networks become more anticorrelated for older groups of participants (Richardson et al., 2018), we may conjecture that those ROIs constitute separate brain networks. In previous studies, it has been uncovered that dPCC is related to the encoding of self-referential pain catastrophizing (Gracely et al., 2004; Lee et al., 2018), whereas V1 is known to be specialized at visual information processing. On the other hand, the other brain regions including RTPJ, LTPJ, and dPCC may constitute a single functional brain network because the connectivity between them is strong in all the age groups. As indicated by Jacoby et al. (2016), these three brain regions are closely related as a ToM network which is associated with the development of reasoning about others’ minds.

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Appendix A. Technical details: copula-DCC-GARCH model for time-varying correlation

Let \( x_t = (x_{t1}, ..., x_{tm}) \) denote the vector of the observed fMRI data at time \( t \). We assume that the joint distribution can be represented by a copula function with time-varying parameters as

\[
F(x_t; \mu_t, \sigma_t, \Theta_t) = C(F(x_{t1}; \mu_{t1}, \sigma_{t1}), ..., F(x_{tm}; \mu_{tm}, \sigma_{tm})|\Theta_t),
\]

where \( F_t \) is the marginal CDF, \( C \) is the copula, and \( \Theta_t \) is a set of time-varying parameters of the copula. We suppose that the conditional mean, \( \mu_{th} \), of the marginal CDF \( F_t \) follows the ARMA(1,1) model, and the conditional variance, \( \sigma_{th} \), follows the GARCH(1,1) model, which can be expressed as

\[
x_{th} = \mu_{th} + \epsilon_{th}, \quad \epsilon_{th} = \sqrt{\sigma_{th}} \epsilon_{th},
\]

with

\[
\mu_{th} = \mu_i + \theta_1 (x_{t-1} - \mu_i) + \theta_2 x_{t-1},
\]

\[
\sigma_{th} = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1},
\]

where \( \epsilon_{th} \) is a sequence of independent random variables having the (reparameterized) Johnson’s SU distribution, \( \eta_t \sim JSU(\mu = 0, \sigma = 1, \nu, \kappa) \) (Ghalanos, 2015; Stasinopoulos et al., 2018). The four parameters \( \mu, \sigma, \nu, \kappa \) represent the mean, standard deviation, skewness, and kurtosis of the distribution. The mean model was chosen to have ARMA(1,1) in order to explain low-frequency oscillations in the mean BOLD signal.

The elliptic copulas employed in this study include time-varying correlations as their parameters. From Eqs. (3) and (4), the density function for the Gaussian copula and the Student’s t-copula with \( \tau \) degrees of freedom can be written as \( c_{gt}(u_{t1}, ..., u_{tm}|R_t, \tau) \) and \( c_{gt}(u_{t1}, ..., u_{tm}|R_0, \tau) \), where
\[ u_{ij} = \tilde{F}(x_{ij}, \mu, \Sigma) \]  
(A.5)

is the probability integral transform of \( x_{ij} \) by the marginal distribution \( F_i \) estimated by fitting the ARMA(1,1)-GARCH(1,1) model. After all, the joint density function of \( x_{ij} \) can be written as, for the Gaussian copula,

\[
f(x_{ij}, \varrho, \mathbf{R}) = 
\left( \mathbf{c}_i(u_{ij}, ..., u_{ij}(R)) \right) \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \left( 1 - \Phi(y_{ij}) \right),
\]

and for the Student's t-copula with \( t \) degrees of freedom,

\[
f(x_{ij}, \varrho, \mathbf{R}, \tau) = 
\left( \tau \left( y_{ij}, ..., y_{ij}(R) \right) \right) \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \left( 1 - \Phi(y_{ij}) \right),
\]

where \( f(y_{ij}) \) is the density function of \( \text{JSU}(0, 1, 1, \nu_i, \nu_j) \).

References


