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Linear time-varying regression with a DCC-GARCH model for volatility

Jong-Min Kim, Hojin Jung and Li Qin
University of Minnesota at Morris, Morris, MN 56267, USA

ABSTRACT
This article provides a new linear state space model with time-varying parameters for forecasting financial volatility. The volatility estimates obtained from the model by using the US stock market data almost exactly match the realized volatility. We further compare our model with traditional volatility models in the ex post volatility forecast evaluations. In particular, we use the superior predictive ability and the reality check for data snooping. Evidence can be found supporting that our simple but powerful regression model provides superior forecasts for volatility.

KEYWORDS
Volatility; time-varying parameter; forecasting; DCC-GARCH

JEL CLASSIFICATION
C32; C53; G10

I. Introduction
Forecasting stock market volatility has been a challenging task. While numerous models have been proposed, no consensus exists to what type of volatility model is the best fit for financial markets. The autoregressive conditional heteroscedastic (ARCH) and generalized ARCH (GARCH) models introduced by Engle (1982) and Bollerslev (1986), respectively, take into account conditionally heteroscedastic innovations in the financial time series. In a similar vein, Orbe, Ferreira and Rodriguez-Poo (2005) employ the AR-parameters allowed to depend on time, while they assume a constant variance. Nevertheless, such a model does not reconcile what we have noted in the financial market, which is that the volatility of stock returns changes over time, and daily volatility can also fluctuate significantly. From this point of view, a model with the time-varying variance function has recently received substantial attention in the literature on forecasting volatility. For example, Fryzlewicz Sapatinas and Rao (2006) use a model in which the AR-parameters are assumed to be constant, and the variance is time-dependent. Chandler and Polonik (2006) propose a model discriminating or clustering different seismic events of locally stationary AR processes with a time-varying variance function.¹

In this study, we aim to introduce an alternative tool for predicting stock return volatility based on the idea of a simple linear regression. We build on a linear state space model for time-varying parameters and extend it by employing a dynamic conditional correlation GARCH (DCC-GARCH) model. Thus, our model allows us to capture not only the time of the transition and structural changes but also the structural dynamic relations in the financial market over time. A number of pairwise comparisons such as the mean square error (MSE) and mean absolute error (MAE) are employed to evaluate the forecast accuracy of our model. In addition, we use more robust tests such as the superior predictive ability (SPA) and the reality check (RC) for data snooping to investigate the forecast performance. This study finds empirical evidence that the new alternative provides the best performance among all the candidates, such as standard GARCH and nonlinear asymmetric GARCH (NAGARCH) models.

The layout of the article is as follows. Section II is devoted to the literature survey of competing models.

¹Chandler and Polonik (2012) model propose the nonstationary AR model with nonconstant unconditional variances. They refer to such a time-varying AR processes as:

\[ Y_t = \sum_{j=1}^{p} \theta_j Y_{t-j} = \sigma \epsilon_t \]

where \( \theta_j \) is the AR parameter functions, \( p \) the order of the model, \( \sigma \) volatility and \( \epsilon_t \) independent identically distributed \( N(0,1) \).
volatility models used to access the performance of our alternative approach. In Section III, our model specification and estimation scheme are provided. Section IV discusses empirical analysis. Concluding remarks are presented in Section V.

II. Volatility models

In this section, we summarize two different traditional volatility models used in this study in order to evaluate the relative efficiency of our model. For a log return series \( r_t = \log \left( \frac{S_t}{S_{t-1}} \right) \), we let \( a_t = r_t - \mathbb{E}_{t-1}[r_t] \) be the innovation at time \( t \). Then \( a_t \) follows a GARCH \((p,q)\) model if

\[
\begin{align*}
 a_t &= \sigma_t \varepsilon_t \\
 h_t^2 &= \alpha_0 + \sum_{i=1}^q \alpha_i a_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}^2
\end{align*}
\]

where \( \{\varepsilon_t\} \) is a sequence of independent and identically distributed random variables with mean 0 and variance 1, \( \alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0 \) and \( \max(p,q) \sum_{i=1}^\infty (\alpha_i + \beta_j) < 1 \). All members of the family of GARCH models can be obtained from a transformation of the conditional SD, \( h_t \), determined by the transformation \( f(\cdot) \) of the innovations, \( a_t \) and lagged transformed conditional SDs.\(^2\)

The standard GARCH model assumes that positive and negative error terms have a symmetric effect on volatility, which means that good and bad news have the same effect on the volatility in the standard GARCH model. But, this assumption is easily violated in the financial market, in particular, by stock returns, in that the negative change in the stock market has a higher impact on the volatility index than a positive change, or vice versa. This was first called leverage effect in the study by Black (1976). As such, the asymmetric GARCH models were developed for accommodating a leverage effect.

A NAGARCH\((p,q)\) model, known as one of the popular asymmetric GARCH models, was introduced by Engle and Ng (1993):

\[
h_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i (a_{t-i} - \gamma h_{t-i})^2 + \sum_{j=1}^p \beta_j h_{t-j}^2
\]

where \( \alpha_0 > 0, \alpha_i \geq 0, \gamma > 0, \) and \( \beta_j \geq 0 \) for \( j = 1, \ldots, p \). In this model, the news impact curve is shifted to the right by the distance \( \gamma h_{t-i} \), and the parameter \( \gamma \) for stock returns is usually estimated to be positive. It reflects the leverage effect, signifying that negative returns increase future volatility by a larger amount than positive returns of the same magnitude.

III. Volatility by using a linear state space regression model with time-varying parameters

We consider a linear state space regression model combined with a DCC-GARCH model in order to predict volatility. Our idea about the new volatility model originates from the simple concept of a linear regression model. Given the linear regression model with two variables \( X \) and \( Y \), we can formally express the estimate of the regression slope coefficient as:

\[
\hat{\beta}_1 = \hat{\rho} \times \frac{\text{sample standard deviation of } Y}{\text{sample standard deviation of } X}
\]

where \( \hat{\rho} \) is the sample correlation between \( X \) and \( Y \). Thus, Equation 1 can be easily rewritten as follows:

\[
\hat{\sigma}_Y = \hat{\beta}_1 \times \frac{\hat{\sigma}_X}{\hat{\rho}}
\]

In our analysis, we can obtain volatility estimates of the S&P 500 index returns \((Y)\) under the time varying context using the expression:

\[
\hat{\sigma}_{Y_i} = \hat{\beta}_1 \times \frac{\hat{\sigma}_{X_i}}{\hat{\rho}_i}
\]

where \( \hat{\sigma}_{X_i} \) is the \( \sqrt{\hat{\sigma}^2} \) of an individual firm stock index and \( \hat{\beta}_i \) is the parameter estimates from a linear state space regression model. \( \hat{\rho}_i \) is a dynamic conditional correlation estimate between the S&P 500 index and the individual firm stock index.

For the estimation of our regression model, we select the daily returns on the S&P 500 and actively traded US firms such as Apple Inc., Amazon.com, IBM, General Electric and Walmart. The sample consists of 3378 daily stock returns over the period from 2 January 2002 to 3 June 2015.\(^3\) All stock index returns...
Table 1. Pearson’s sample correlation coefficients (ρ).

<table>
<thead>
<tr>
<th></th>
<th>Apple</th>
<th>Amazon</th>
<th>IBM</th>
<th>GE</th>
<th>Walmart</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>0.8043</td>
<td>0.8008</td>
<td>0.6770</td>
<td>0.6027</td>
<td>0.7778</td>
</tr>
</tbody>
</table>

prices are retrieved from http://quote.yahoo.com. We calculate Pearson’s sample correlation coefficient \( \hat{\rho} \) between S&P 500 index and each individual firm’s stock index in order to check whether we consider the appropriate firms to estimate the volatility of the S&P 500. Table 1 clearly shows that all sample coefficients are close to 1, implying a strong correlation between the two indices. The larger the magnitude of the coefficient, the stronger the relationship. For example, the sample correlation between the S&P 500 and Apple index returns is 0.8043, showing that these two indices have the strongest relationship among our sample firms.

Primiceri (2005) introduces the time-varying parameter vector autoregressive (TVP-VAR) model, combined with stochastic volatility. According to Nakajima (2011), the TVP-VAR model has an advantage over the constant parameter VAR model in that it is able to capture the structural changes without wasting useful observations from the sample. Nakajima (2011) and Jebabli, Arouri and Teulon (2014) estimate the TVP-VAR model with stochastic volatility, by employing the Markov Chain Monte Carlo (MCMC) method for the Japanese macroeconomic variables and food commodities returns, respectively.

To obtain the estimates of the time-varying regression slopes, we use a linear time-varying regression (LTVR) model, which is similar to the TVP-VAR model but provides more flexibility with a parsimonious specification. Following Zivot and Yollin (2012), we define LTVR as:

\[
\begin{align*}
\text{Measurement equation:} & & y_t &= \alpha_t + \beta_t x_t + \nu_t, & \nu_t &\sim N(0, \sigma^2_{\nu}) \\
\text{Transition equation:} & & \alpha_t &= \alpha_{t-1} + \omega_{\alpha,t}, & \omega_{\alpha,t} &\sim N(0, \sigma^2_{\alpha}) \\
\text{Transition equation:} & & \beta_t &= \beta_{t-1} + \omega_{\beta,t}, & \omega_{\beta,t} &\sim N(0, \sigma^2_{\beta})
\end{align*}
\]

where \( \sigma^2_{\nu} \) indicates the variance of the observation noise. The first diagonal element of the variance matrix of the system noise is \( \sigma^2_{\alpha} \), and \( \sigma^2_{\beta} \) is placed in the second diagonal element. The measurement equation has the time-varying coefficients (\( \beta_t \)). The variance of disturbance \( \sigma^2_{\nu} \) is also assumed to be time-variant. In this study, the parameters \( \beta_t \) follow the first-order random walk process. This assumption allows both the temporary and permanent effects in the parameters. As an alternative setup to the TVP-type models, both the parameters, \( \alpha_t \) and \( \beta_t \), are designed to vary over time. This is one of the great extensions in our model specification, because the added dimension makes our model more flexible.

If \( y_t \) is an \((m \times 1)\) time series, then the model given by Equation 3 can be rewritten in a conventional state space form as follows:

\[
\begin{align*}
\mathbf{y}_t &= \mathbf{F}_t \mathbf{\theta}_t + \mathbf{v}_t \\
\mathbf{\theta}_t &= \mathbf{G}_t \mathbf{\theta}_{t-1} + \mathbf{w}_t
\end{align*}
\]

where \( \mathbf{F}_t \) is an \((m \times p)\) vector of time-varying covariates; \( \mathbf{\theta}_t \) is a \((p \times 1)\) non-stationary state vector, defined as \( \mathbf{\theta}_t = (\alpha_t, \beta_t)' \) and \( \mathbf{v}_t \) and \( \mathbf{w}_t \) are \((m \times 1)\) matrices of independently and identically distributed normal random variables with mean 0 and variances \( \mathbf{V}_t \) and \( \mathbf{W}_t \), respectively. Notice that the system matrices are

\[
\mathbf{F}_t = \begin{pmatrix} 1 & x_t \end{pmatrix}, \quad \mathbf{V}_t = \sigma^2_{\nu}, \quad \mathbf{G}_t = \mathbf{I}_p, \quad \text{and} \quad \mathbf{W}_t = \begin{pmatrix} \sigma^2_{\alpha} & 0 \\ 0 & \sigma^2_{\beta} \end{pmatrix}
\]

We assume that the measurement disturbances \( v_t \) and the transition disturbances \( w_t \) are mutually independent. The initial distribution is

\[
\begin{align*}
\text{Regression:} & & y_t &= \mathbf{x}_t \beta + \mathbf{z}_t \alpha_t + \epsilon_t, & \epsilon_t &\sim N(0, \sigma^2_{\epsilon}) \\
\text{Time-varying coefficients:} & & \alpha_{t+1} &= \alpha_t + u_t, & u_t &\sim N(0, \Sigma), \quad t = 0, \ldots, n-1 \\
\text{Stochastic volatility:} & & \sigma^2_{t+1} &= \exp(h_t), & h_{t+1} &= \phi h_t + \eta_t, & \eta_t &\sim N(0, \sigma^2_{\eta}), \quad t = 0, \ldots, n-1
\end{align*}
\]

where \( y_t \) is a scalar of an observable variable, \( \mathbf{x}_t \) and \( \mathbf{z}_t \) are vectors of covariates, \( \beta \) is a vector of constant coefficients, \( \alpha_t \) is a vector of time-varying coefficients, and \( h_t \) is stochastic volatility.

\[\text{If the constant volatility is assumed here, the parameters can be estimated by using the Kalman filter. For a discussion of the estimation methodology, see Harrison and West (1997).}\]
\( \theta_0 \sim N(m_0, C_0) \) where \( m_0 = 0, \)
\( C_0 = k \times I_2, \) \( k = 10^7 \) \( \tag{4} \)

where \( m_0 \) is the expected value of the pre-sample state vector and \( C_0 \) is the variance matrix of the pre-sample state vector.\(^6\)

Let \( \psi \) denote parameters embedded in the system matrices defined above. For the state space model with a fixed value of \( \psi \), the Kalman filter produces the prediction errors. The parameter can be estimated from the data \( \{y_t\} \) using the prediction error decomposition of the log-likelihood function:

\[ \hat{\psi}_{MLE} = \arg \max_{\psi} \ln L(\psi|y_t) = \sum_{t=1}^{T} \ln f(y_t|I_{t-1}; \psi) \]

where \( f(y_t|I_{t-1}; \psi) \) is the conditional density of \( y_t \) given information available at time \( t-1 \). In our study, the parameter estimation is accomplished via Kalman filtering functions, \texttt{dlm.MLE}, in the R package \texttt{dlm}.

After the parameters estimation, we are interested in the dynamic conditional correlation parameters. We estimate a DCC-GARCH model for which correlation varies over time. This model has a two-stage algorithm to estimate the parameters. In the first stage, the conditional volatility \( \sigma_t \) is estimated by using a univariate GARCH model. The second stage accounts for dynamics in the correlation given the parameters from the first stage.\(^7\) The dynamic conditional correlation structure is:

\[ Q_t = \frac{1}{T} \sum_{t=1}^{T} v_t v_t' + \alpha \left( v_{t-1} v_{t-1}' - \frac{1}{T} \sum_{t=1}^{T} v_t v_t' \right) \]
\[ + \beta \left( Q_{t-1} - \frac{1}{T} \sum_{t=1}^{T} v_t v_t' \right) \] \( \tag{5} \)

where \( v_t = \frac{\varepsilon_t - \mu}{\sigma_t} \). Note that \( \mu \) is a vector of expected returns. In this study, each off-diagonal element in the variance-covariance matrix \( Q_t \) is the correlation between the S&P 500 index and a selected firm’s stock market index at time \( t \). We use the \texttt{dcc.estimation} command in the R package \texttt{ccgarch} to estimate each conditional correlation. Model selection criteria such as Schwarz Information Criterion (SIC) and Akaike Information Criterion (AIC) are used to determine the appropriate model. In particular, the GARCH(1,1) specification is employed in this study. Numerous empirical studies support the superiority of the GARCH(1,1) model that best fits financial time series.

\section*{IV. Forecast evaluation}

\textbf{An illustrated example with forecast measures}

Figure 1 plots the long historical series of S&P 500 index changes. The S&P 500 index has had a gentle upward tendency over time. In fact, the annualized return for the S&P 500 index over the time span was approximately 8.6%. However, we can easily identify two crash time periods. The first one corresponds to the 2002 dot-com bubble burst, while the second one

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{snp500.png}
\caption{S&P 500 index from 2002 to 2015.}
\end{figure}

\(^6\)For a detailed description of the estimation procedure for the dynamic linear models, see Petris, Petrone, and Campagnoli (2009).

\(^7\)Engle and Sheppard (2001) provide a comprehensive discussion on a DCC-GARCH model. See http://vlab.stern.nyu.edu/doc/13/topic=mdls for more details.
occurred on September 2008, the period for the financial crisis.

Figure 2 shows the parameter estimates, the estimated conditional correlation coefficients and a comparison between the estimated volatilities using the model and the realized volatilities using the daily squared returns $r_t^2$.\(^8\)

The sign for the time-varying parameters ($\beta_t$) is positive during most of the sample periods, meaning that these selected firms’ stock returns could positively and significantly affect S&P 500 stock market returns.\(^9\) We also estimate the DCC-GARCH(1,1) model. The overall values of the conditional correlations are lower than those of the Pearson’s sample correlation coefficients. Not surprisingly, the conditional correlation coefficients have similar patterns as that of the betas over time. For instance, the correlation estimates between S&P 500 and Apple or Walmart stock returns also change dramatically like the betas for those firms.

The volatility of stock returns has been remarkably stable, with the exception of the financial crisis. The stock market volatility was significantly higher during these periods. As we can see from the figure, however, the volatility seen at that time was relatively short-lived. When judged by the volatility based on the squared daily returns, our volatility model provides seemingly good forecasts. For instance, the lower left panel displays that there is a substantially synchronous match in volatilities from the LTVR model using the Apple stock index and the realized squared daily returns. This visual impression consistently occurs across the leading companies.

Every lower right panel shows that the differences of these two volatilities mostly stay between −50 and 50; however, a regional peak emerged during the financial crisis. The difference is primarily responsible for the fact that our model takes into account the possibility of the dynamic structure changes but the proxy does not. Thus, our model with time-varying parameters detects the transition time and the asymmetric effects in the stock market. One salient feature of our empirical results is that given a lower value of the Pearson’s correlation coefficient, we see a higher accuracy of the volatility from our proposed model. While Figure 2 seems to suggest that our model provides superior forecasting performance, we also need to be cautious in interpreting it, because the squared return is a proxy for the underlying latent volatility. Therefore, we carry out empirical exercises to examine the performance of our volatility specification against competing models.

We use four statistical loss functions to evaluate the forecast accuracy, such as

$$\text{MSE}_1 = n^{-1} \sum_{t=1}^{n} (\sigma_t - h_t)^2$$
$$\text{MSE}_2 = n^{-1} \sum_{t=1}^{n} (\sigma_t^2 - h_t^2)$$
$$\text{MAE}_1 = n^{-1} \sum_{t=1}^{n} |\sigma_t - h_t|$$
$$\text{MAE}_2 = n^{-1} \sum_{t=1}^{n} |\sigma_t^2 - h_t^2|$$

These functions with mathematical simplicity are the popular measures to evaluate forecasting performance in the literature (e.g. Hansen and Lunde (2005) and West and Cho (1995)). In the loss functions, we still use the intraday squared returns $r_t^2$ as a proxy for the latent volatility $\sigma_t^2$. The common practice in the literature justifies the use of the squared return as a proxy for volatility (Pagan and Schwert (1990) and West and Cho (1995)).

To test the forecasting power of our model, we include the popular symmetric and asymmetric GARCH models in the competing candidates in this empirical study. As one of our competing models, the standard GARCH is considered because of its ability to detect structure changes of volatility. However, a symmetric model could be associated with the drawback that it is unable to capture a leverage or asymmetric effect. For this reason, we also consider one of the asymmetric GARCH models as an alternative in this study. In particular, the NAGARCH model is selected among the family of asymmetric GARCH processes based on the AIC and BIC selection criteria. Therefore, the corresponding volatility forecast $h_t$ is obtained from three different volatility models.

---

\(^8\)As pointed out by an anonymous reviewer, the squared return as a proxy for the volatility may be inefficient because of its high variance or low precision. But, the squared return is an unbiased estimator of the volatility (Andersen and Bollerslev 1998). The aim of this study is to verify whether our proposed new method provides more accurate forecasts of volatility than other competing volatility models. To do this, we proxy the squared return, which provides an unbiased estimate for the latent volatility. The use of squared returns models is more common in the time series volatility models; see Poon and Granger (2003) for an extensive review and references.

\(^9\)Note that a 95% confidence interval is incorporated into the panel.
Table 2 presents the results from the model comparisons. From the selected firms’ return data, it is evident that the LTVR model provides the most accurate forecasts in terms of all loss functions, with the possible exception of the MSE criterion. The empirical relative efficiency (RE) is defined as the ratio of the variances of the forecasting errors obtained from our model
and the better competing model given alternatives. It is worth mentioning that the standard GARCH model, which shows one of the best sample performances in our empirical analysis is at least 9.91% less accurate than our proposed model from the Walmart return data in terms of the $\text{MAE}_2$ loss function. The performance of the LTVR model is robust under other firms’ returns.

Figure 2. Continued.
In this subsection, we employ the more robust tests such as Hansen’s (2005) superior predictive ability (SPA) and White’s (2000) reality check (RC) for data snooping to evaluate whether an exhibited superiority in performance of our proposed method in the previous subsection is significant or could have occurred by chance. We implement the stationary bootstrap with 1000 replications and a window size 12. More details of this procedure are described in (2005). We consider two alternative volatility models, the standard GARCH and NAGARCH, which are notated as \(k = 1, 2\), respectively. In this analysis, the LTVR model is taken as the benchmark model. For each model, we generate the relative performance variable based on the loss function, which is defined as

\[
D_{k,t} = \phi(\xi_t, h_{k,t-1}) - \phi(\xi_t, h_{t-1}), \quad k = 1, 2
\]

where \(\phi(\xi_t, h_{k,t-1}) = (\xi_t - h_{k,t-1})^2\); \(\xi_t\) represents squared returns as a volatility proxy, and \(r^2; h_{k,t-1}\) is a volatility forecast of the \(k\)th forecasting model based on \(t - 1\) information. Assuming that \(D_k = E(D_{k,t})\) is well defined, the null hypothesis in this analysis is that

\[
H_0 : \mu \leq 0
\]

Table 2. Forecast evaluation.

<table>
<thead>
<tr>
<th></th>
<th>MSE_1</th>
<th>MSE_2</th>
<th>MAE_1</th>
<th>MAE_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple, Inc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTVR</td>
<td>0.6887</td>
<td>25.3849</td>
<td>0.5639</td>
<td>1.5785</td>
</tr>
<tr>
<td>GARCH</td>
<td>2.8591</td>
<td>41.9237</td>
<td>1.5151</td>
<td>4.7106</td>
</tr>
<tr>
<td>NAGARCH</td>
<td>2.8992</td>
<td>46.0080</td>
<td>1.5127</td>
<td>4.7836</td>
</tr>
<tr>
<td>RE (%)</td>
<td>75.91</td>
<td>39.45</td>
<td>62.78</td>
<td>66.66</td>
</tr>
<tr>
<td>Amazon.com</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTVR</td>
<td>0.9914</td>
<td>105.0816</td>
<td>0.6250</td>
<td>2.3004</td>
</tr>
<tr>
<td>GARCH</td>
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<td>70.2007</td>
<td>1.8865</td>
<td>6.6055</td>
</tr>
<tr>
<td>NAGARCH</td>
<td>4.1977</td>
<td>69.2504</td>
<td>1.8758</td>
<td>6.5485</td>
</tr>
<tr>
<td>RE (%)</td>
<td>76.38</td>
<td>-51.74</td>
<td>66.68</td>
<td>64.67</td>
</tr>
<tr>
<td>IBM</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>LTVR</td>
<td>0.4617</td>
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<tr>
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<td>25.4831</td>
<td>0.8127</td>
<td>2.0953</td>
</tr>
<tr>
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<td>1.0015</td>
<td>23.8611</td>
<td>0.8020</td>
<td>2.0640</td>
</tr>
<tr>
<td>RE (%)</td>
<td>53.90</td>
<td>12.14</td>
<td>41.60</td>
<td>33.45</td>
</tr>
<tr>
<td>GE</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>LTVR</td>
<td>0.3219</td>
<td>15.2375</td>
<td>0.3942</td>
<td>1.0528</td>
</tr>
<tr>
<td>GARCH</td>
<td>1.5946</td>
<td>39.3869</td>
<td>0.9696</td>
<td>2.9634</td>
</tr>
<tr>
<td>NAGARCH</td>
<td>1.6070</td>
<td>41.9505</td>
<td>0.9603</td>
<td>2.9882</td>
</tr>
<tr>
<td>RE (%)</td>
<td>79.81</td>
<td>61.31</td>
<td>58.95</td>
<td>64.47</td>
</tr>
<tr>
<td>Walmart</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTVR</td>
<td>0.6779</td>
<td>28.2098</td>
<td>0.5583</td>
<td>1.5949</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.8494</td>
<td>24.3230</td>
<td>0.7176</td>
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<tr>
<td>NAGARCH</td>
<td>0.8442</td>
<td>24.1243</td>
<td>0.7156</td>
<td>1.7641</td>
</tr>
<tr>
<td>RE (%)</td>
<td>19.70</td>
<td>-16.94</td>
<td>21.98</td>
<td>9.91</td>
</tr>
</tbody>
</table>

Superiority prediction tests

In this subsection, we employ the more robust tests such as Hansen’s (2005) superior predictive ability (SPA) and White’s (2000) reality check (RC) for data snooping to evaluate whether an exhibited superiority in performance of our proposed method in the previous subsection is significant or could have occurred by chance. We implement the stationary bootstrap with 1000 replications and a window size 12. More details of this procedure are described in (2005). We consider two alternative volatility models, the standard GARCH and NAGARCH, which are notated as \(k = 1, 2\), respectively. In this analysis, the LTVR model is taken as the benchmark model. For each model, we generate the relative performance variable based on the loss function, which is defined as

\[
D_{k,t} = \phi(\xi_t, h_{k,t-1}) - \phi(\xi_t, h_{t-1}), \quad k = 1, 2
\]

where \(\phi(\xi_t, h_{k,t-1}) = (\xi_t - h_{k,t-1})^2\); \(\xi_t\) represents squared returns as a volatility proxy, and \(r^2; h_{k,t-1}\) is a volatility forecast of the \(k\)th forecasting model based on \(t - 1\) information. Assuming that \(D_k = E(D_{k,t})\) is well defined, the null hypothesis in this analysis is that

\[
H_0 : \mu \leq 0
\]
Tests for reality check and superior predictive ability.

<table>
<thead>
<tr>
<th>p-value</th>
<th>Apple</th>
<th>Amazon</th>
<th>IBM</th>
<th>GE</th>
<th>Walmart</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC</td>
<td>1.000</td>
<td>0.208</td>
<td>1.00</td>
<td>1.00</td>
<td>0.188</td>
</tr>
<tr>
<td>SPA</td>
<td>1.000</td>
<td>0.198</td>
<td>1.00</td>
<td>1.00</td>
<td>0.172</td>
</tr>
</tbody>
</table>

Note: The table presents p-values of the RC and SPA test.

where $\mu = (D_1, D_2)'$. All essential assumptions of our framework are consistent with (2005). Our null hypothesis is that our proposed model is as good as any of the alternatives. Therefore, when there is no evidence against the null hypothesis, our proposed model significantly outperforms the models currently being used in forecasting.

The estimation results are presented in Table 3. The $p$-values of the RC and SPA tests are provided for each sample firm. The $p$-values clearly show that there is no evidence that the LTVR model is outperformed even if a moderate (10%) significance level is used. The $p$-values do not differ much across the sample firms, which verify that our results are not sensitive. The results show that the LTVR model is clearly preferred, which confirms the earlier findings from Table 2.

V. Conclusion

This article introduces a tractable framework for forecasting volatility and its empirical applications. We consider a linear state space regression model with time-varying parameters and dynamic conditional correlations. In the empirical analysis, we evaluate its performance against conventional volatility models using the US daily stock returns. In particular, we compare forecast errors generated from each model under various assessment measures. The empirical results show that our model outperforms the competing models in terms of its ability to forecast the conditional variance.

Volatility forecast plays a central role in financial markets such as, asset pricing, portfolio selection or market risk management. Thus, it is important that financial market agents consider the volatility model, which provides more accurate forecasts. Our proposed model will be beneficial to the economic agents, including policy makers in the market.

Disclosure statement

No potential conflict of interest was reported by the authors.

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